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Chapter I. Kinematics of Gears

Discussion Map

■ Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another.
■ Most gear drives cause a change in the speed of the output gear relative to the input gear.
■ Some of the most common types of gears are spur gears, helical gears, bevel gears, and worm/wormgear sets.

I.1.Introduction

Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another. The teeth of a driving gear mesh accurately in the spaces between teeth on the driven gear as shown in Figure I–1. The driving teeth push on the driven teeth, exerting a force perpendicular to the radius of the gear. Thus, a torque is transmitted, and because the gear is rotating, power is also transmitted.

- **Speed Reduction Ratio.** Often gears are employed to produce a change in the speed of rotation of the driven gear relative to the driving gear. In Figure I–1, if the smaller top gear, called a *pinion*, is driving the larger lower gear, simply called the *gear*, the larger gear will rotate more slowly. The amount of speed reduction is dependent on the ratio of the number of teeth in the pinion to the number of teeth in the gear according to this relationship:

$$\frac{n_p}{n_G} = \frac{N_G}{N_p} \qquad 1.1$$

The basis for this equation will be shown later in this chapter. But to show an example of its application here, consider that the pinion in Figure 1–1 is rotating at 1800 rpm. You can count the number of teeth in the pinion to be 11 and the number of teeth in the gear to be 18. Then we can compute the rotational speed of the gear by solving Equation (1.1) for n_G :

$$n_G = n_P(N_P/N_G) = (1800 \text{ rpm})(11/18) = 1100 \text{ rpm}$$

When there is a reduction in the speed of rotation of the gear, there is a simultaneous proportional *increase* in the torque transmitted to the shaft carrying the gear. More will be said about this later, also.

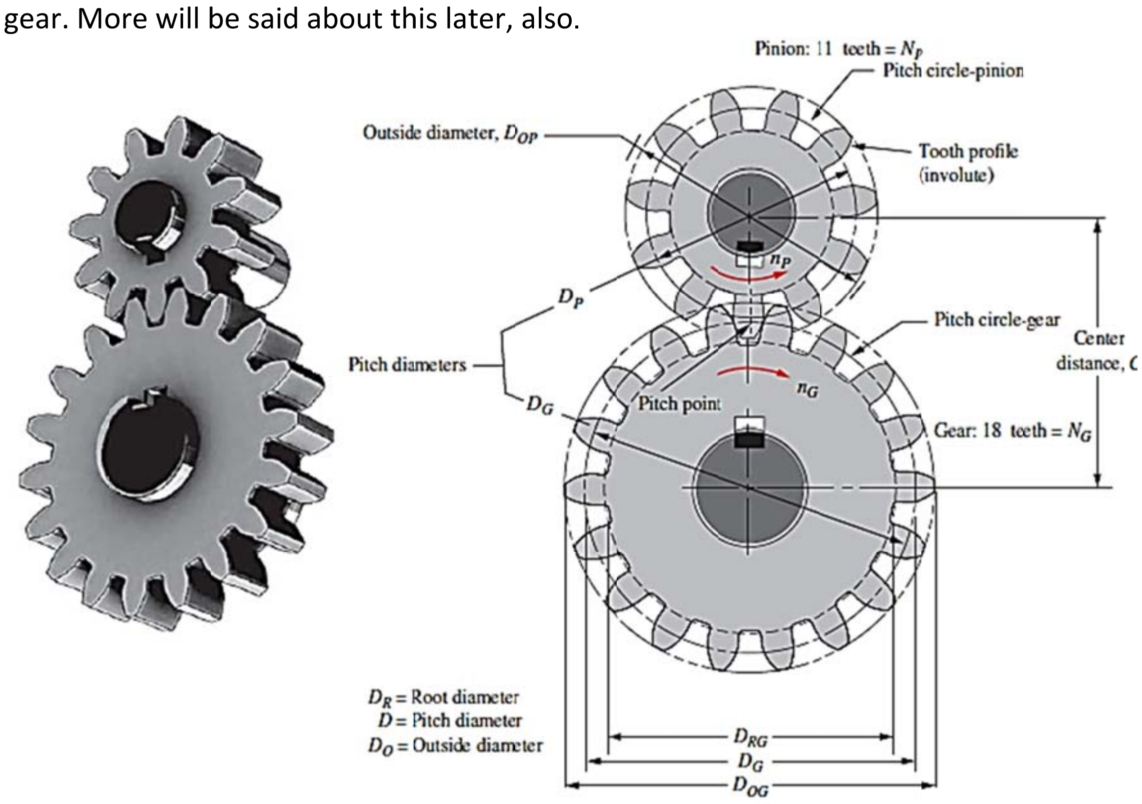

Fig. I–1 Pair of spur gears. The pinion drives the gear.

- **Kinds of Gears.** Several kinds of gears having different tooth geometries are in common use. To acquaint you with the general appearance of some, their basic descriptions are given here. Later we will describe their geometry more completely. Figure I–2 shows a photograph of many kinds of gears. Labels indicate the major types of gears that are discussed in this chapter: *spur gears, helical gears, bevel gears,* and *worm/wormgear sets.* Obviously, the shafts that would carry the gears are not included in this photograph.

❖ *Spur gears* have teeth that are straight and arranged parallel to the axis of the shaft that carries the gear. The curved shape of the faces of the spur gear teeth have a special geometry called an *involute curve,* described later in this chapter. This shape makes it possible for two gears to operate together with smooth, positive transmission of power. Figure I–1 also shows the side view of spur gear teeth, and the involute curve shape is evident there. The shafts carrying the gears are parallel.

❖ The teeth of *helical gears* are arranged so that they lie at an angle with respect to the axis of the shaft. The angle, called the *helix angle*, can be virtually any angle. Typical helix angles range from approximately 10° to 30°, but angles up to 45° are practical. The helical teeth operate more smoothly than equivalent spur gear teeth, and stresses are lower. Therefore, a smaller helical gear can be designed for a given power-transmitting capacity as compared with spur gears. One disadvantage of helical gears is that an axial force, called a *thrust force,* is generated in addition to the driving force that acts tangent to the basic cylinder on which the teeth are arranged. The designer must consider the thrust force when selecting bearings that will hold the shaft during operation. Shafts carrying helical gears are typically arranged parallel to each other. However, a special design, called *crossed helical* gears, has 45° helix angles, and their shafts operate 90° to each other.

- *Bevel gears* have teeth that are arranged as elements on the surface of a cone. The teeth of straight bevel gears appear to be similar to spur gear teeth, but they are tapered, being wider at the outside and narrower at the top of the cone. Bevel gears typically operate on shafts that are 90° to each other. Indeed, this is often the reason for specifying bevel gears in a drive system. Specially designed bevel gears can operate on shafts that are at some angle other than 90°. When bevel gears are made with teeth that form a helix angle similar to that in helical gears, they are called *spiral bevel gears*. They operate more smoothly than straight bevel gears and can be made smaller for a given power transmission capacity. When both bevel gears in a pair have the same number of teeth, they are called *miter gears* and are used only to change the axes of the shafts to 90°. No speed change occurs.

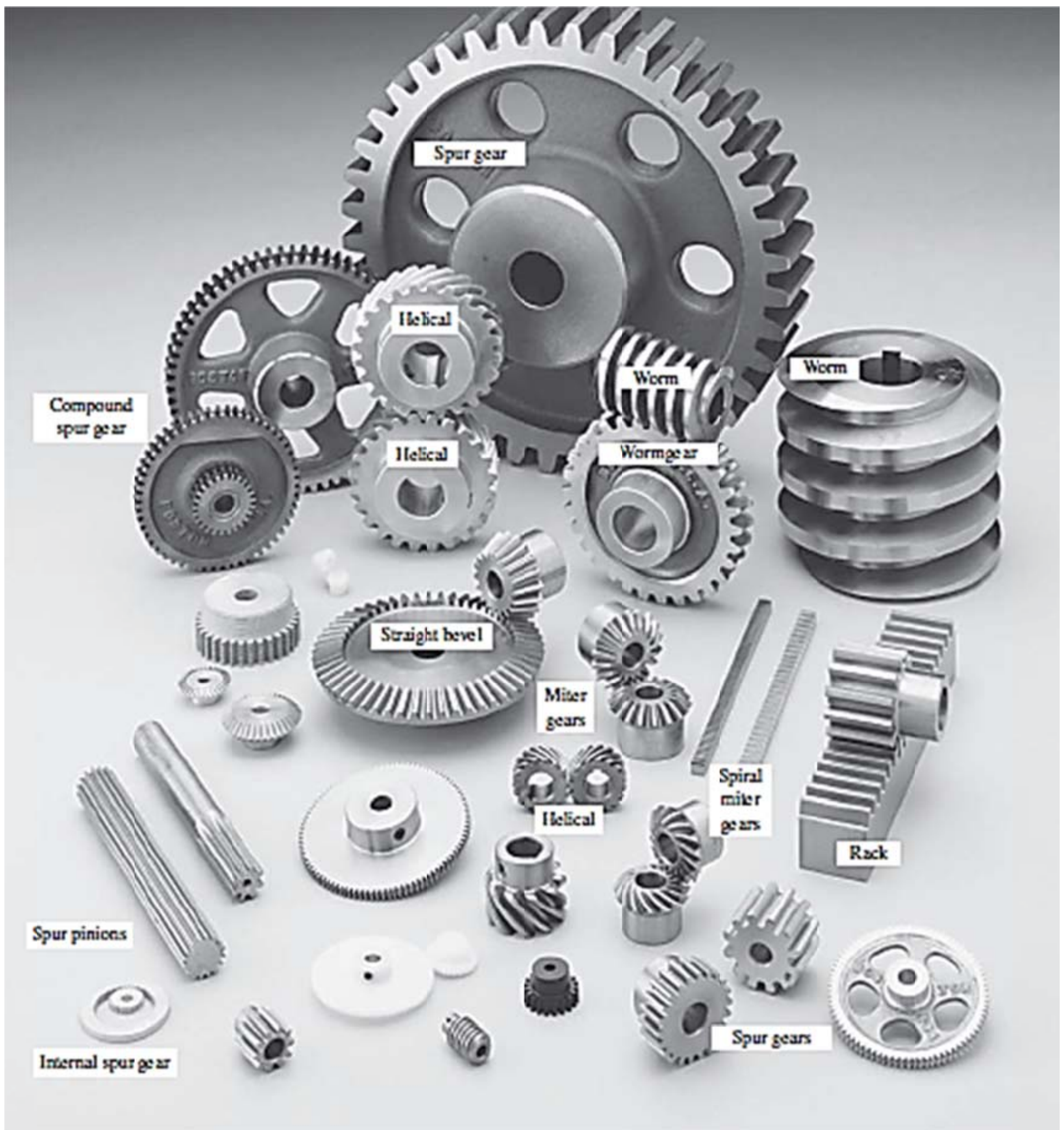

Fig. I–2 A variety of gear types

I.2. Basic Geometry of Spur Gears

The fundamentals of gearing are illustrated through the spur gear tooth, both because it is the simplest, and hence most comprehensible, and because it is the form most widely used, particularly for instruments and control systems. The basic geometry and nomenclature of a spur gear mesh is shown in Figure I–3. The essential features of a gear mesh are:

1. Center distance.
2. The pitch circle diameters (or pitch diameters).
3. Size of teeth (or module).
4. Number of teeth.
5. Pressure angle of the contacting involutes. Details of these items along with their interdependence and definitions are covered in subsequent paragraphs.

I.3. The Law Of Gearing

A primary requirement of gears is the constancy of angular velocities or proportionality of position transmission. Precision instruments require positioning fidelity. High-speed and/or high-power gear trains also require transmission at constant angular velocities in order to avoid severe dynamic problems.

Constant velocity (i.e., constant ratio) motion transmission is defined as "conjugate action" of the gear tooth profiles. A geometric relationship can be derived for the form of the tooth profiles to provide conjugate action, which is summarized as the Law of Gearing as follows: "A common normal to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line of- centers called the pitch point."

Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves.

I.4. The Involute Curve

There is almost an infinite number of curves that can be developed to satisfy the law of gearing, and many different curve forms have been tried in the past. Modern gearing (except for clock gears) is based on involute teeth. This is due to three major advantages of the involute curve:

1. Conjugate action is independent of changes in center distance.
2. The form of the basic rack tooth is straight-sided, and therefore is relatively simple and can be accurately made; as a generating tool it imparts high accuracy to the cut gear tooth.
3. One cutter can generate all gear teeth numbers of the same pitch.

The involute curve is most easily understood as the trace of a point at the end of a taut string that unwinds from a cylinder. It is imagined that a point on a string, which is pulled taut in a fixed direction, projects its trace onto a plane that rotates with the base circle. See Figure I–4. The base cylinder, or base circle as referred to in gear literature, fully defines the form of the involute and in a gear it is an inherent parameter, though invisible.

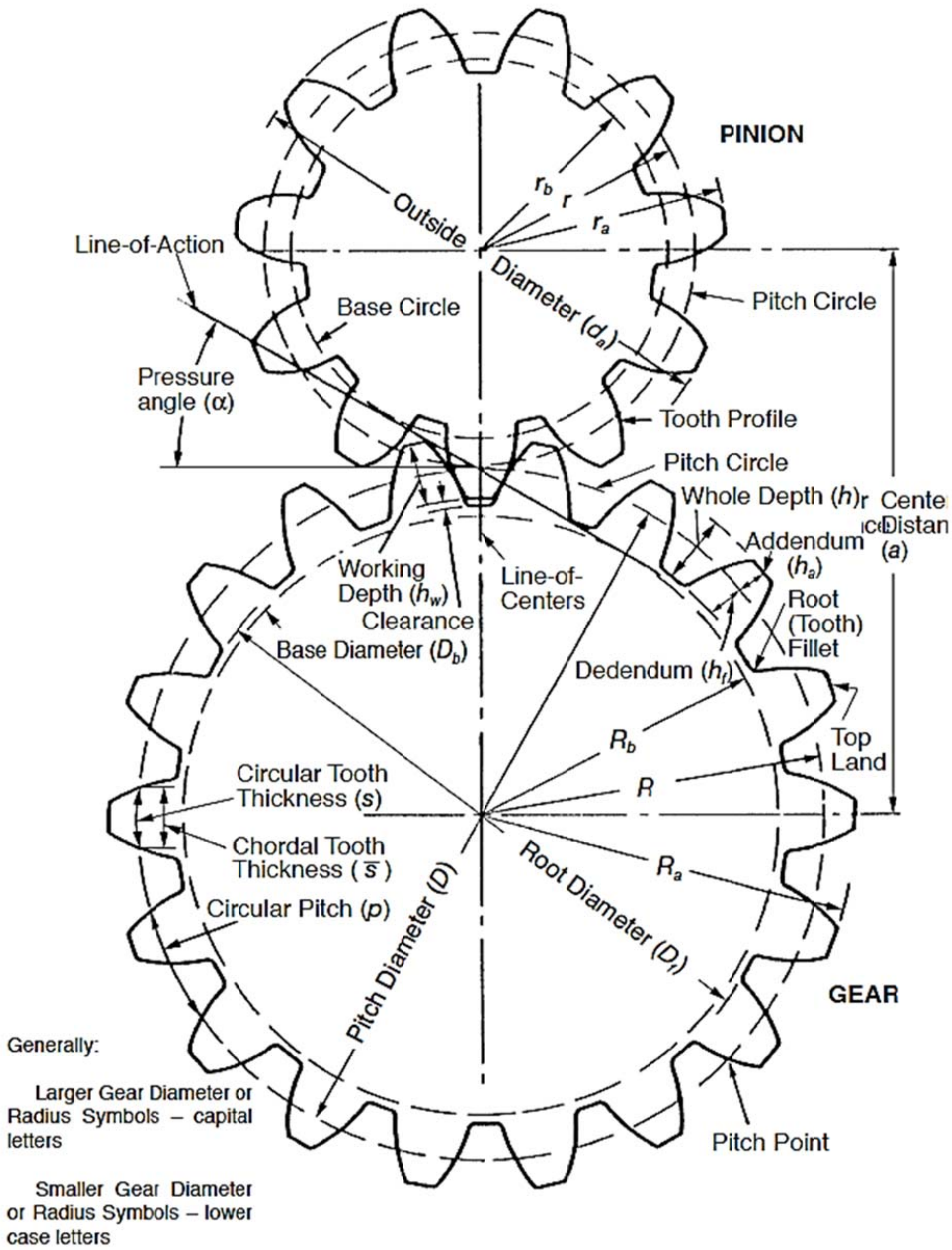

Fig. I–3 Basic Gear Geometry

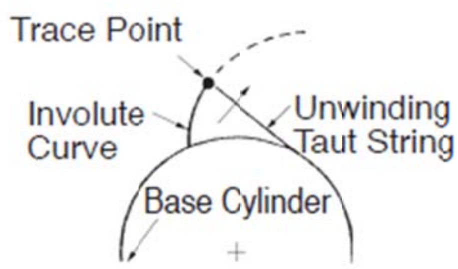

Fig. I–4 Generation of an Involute by a Taut String

The development and action of mating teeth can be visualized by imagining the taut string as being unwound from one base circle and wound on to the other, as shown in Figure I–5a. Thus, a single point on the string simultaneously traces an involute on each base circle's rotating plane. This pair of involutes is conjugate, since at all points of contact the common normal is the common tangent which passes through a fixed point on the line-ofcenters. If a second winding/unwinding taut string is wound around the base circles in the opposite direction, Figure I–5b, oppositely curved involutes are generated which can accommodate motion reversal. When the involute pairs are properly spaced, the result is the involute gear tooth, Figure I–5c.

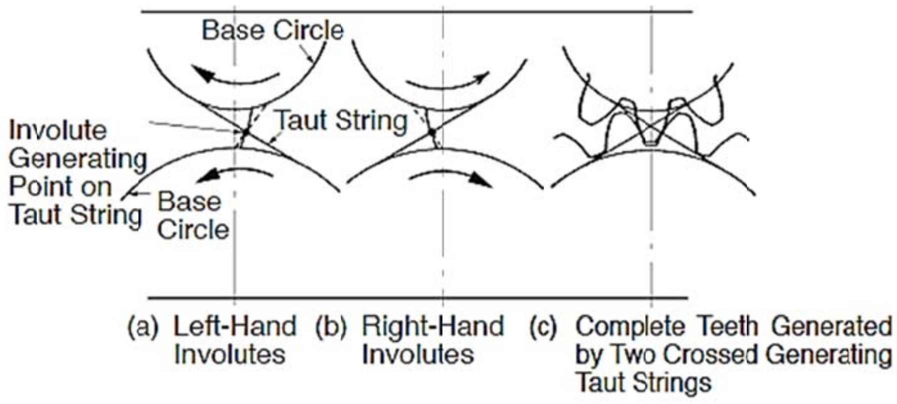

Fig. I–5 Generation and Action of Gear Teeth

I.5. Pitch Circles

Referring to Figure I–6, the tangent to the two base circles is the line of contact, or line-of-action in gear vernacular. Where this line crosses the line-of-centers establishes the pitch point, P. This in turn sets the size of the pitch circles, or as commonly called, the pitch diameters. The ratio of the pitch diameters gives the velocity ratio:

Velocity ratio of gear 2 to gear 1 is:

$$i = \frac{d_1}{d_2}$$

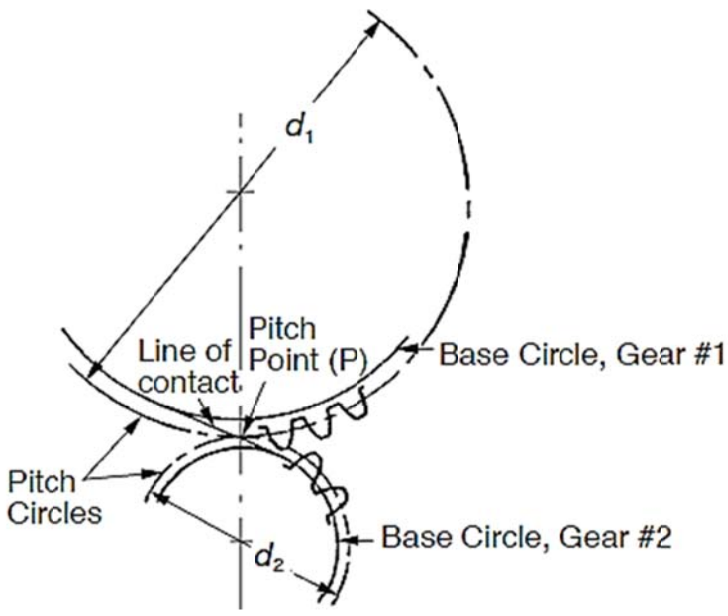

Fig. I–6 Definition of Pitch Circle and Pitch Point

I.6. Pitch And Module

Essential to prescribing gear geometry is the size, or spacing of the teeth along the pitch circle. This is termed pitch, and there are two basic forms.

- ☒ **Circular pitch :** A naturally conceived linear measure along the pitch circle of the tooth spacing. Referring to Figure I–7, it is the linear distance (measured along the pitch circle arc) between corresponding points of adjacent teeth. It is equal to the pitch-circle circumference divided by the number of teeth:

$$p\ circular\ pitch = \frac{pitch\ circle\ circumference}{number\ of\ teeth} = \frac{\pi\, d}{Z}$$

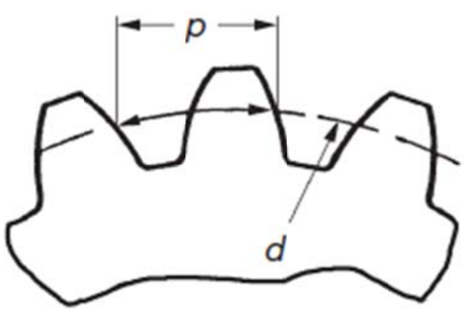

Fig. I–7 Definition of Circular Pitch

- ☒ **Module :** Metric gearing uses the quantity module *m* in place of the American inch unit, diametral pitch. The module is the length of pitch diameter per tooth. Thus:

$$m = \frac{d}{Z}$$

- **Relation of pitches:** From the geometry that defines the two pitches, it can be shown that module and circular pitch are related by the expression:

$$\frac{P}{m} = \pi$$

This relationship is simple to remember and permits an easy transformation from one to the other.

Diametral pitch P_d is widely used in England and America to represent the tooth size. The relation between diametral pitch and module is as follows:

$$m = \frac{25.4}{P_d}$$

I.7. Module Sizes And Standards

Module *m* represents the size of involute gear tooth. The unit of module is mm. Module is converted to circular pitch *p*, by the factor π.

$$P = \pi .\ M$$

Table I-1 is extracted from JIS B 1701-1973 which defines the tooth profile and dimensions of involute gears. It divides the standard module into three series. Figure I–8 shows the comparative size of various rack teeth.

Table I-1 Standard Values of Module unit: mm

Series 1	Series 2	Series 3	Series 1	Series 2	Series 3	Series 1	Series 2	Series 3	Series 1	Series 2	Series 3
0.1		0.65		0.7			3.5			14	
	0.15			0.75		4		3.75	16		
0.2			0.8				4.5			18	
	0.25			0.9		5			20		
0.3			1				5.5			22	
	0.35		1.25			6		6.5	25		
0.4			1.5				7			28	
	0.45			1.75		8			32		
0.5			2	2.25			9		40		
	0.55		2.5			10				36	
0.6							11			45	
	2.75		3		3.25	12			50		

Circular pitch, *p*, is also used to represent tooth size when a special desired spacing is wanted, such as to get an integral feed in a mechanism. In this case, a circular pitch is chosen that is an integer or a special fractional value. This is often the choice in designing position control systems. Another particular usage is the drive of printing plates to provide a given feed.

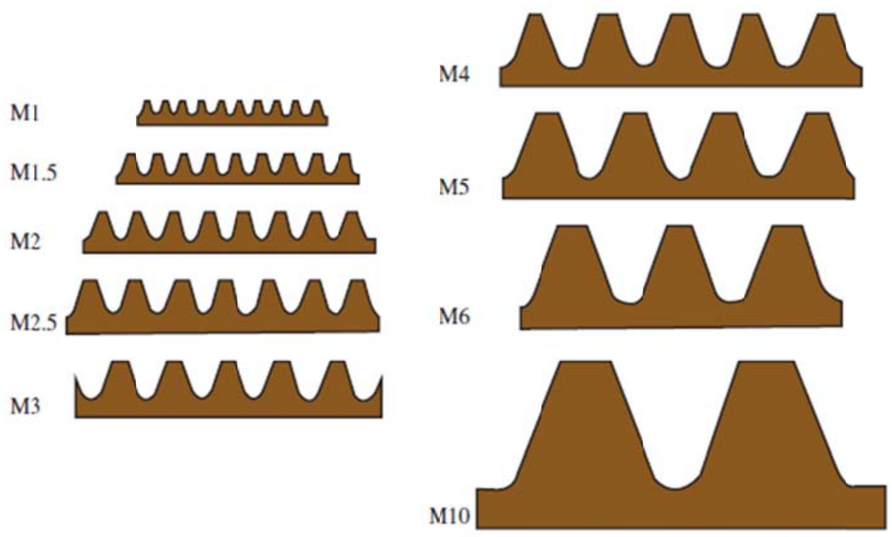

Fig. I–8 Comparative Size of Various Rack Teeth

Most involute gear teeth have the standard whole depth and a standard pressure angle α = 20°. Figure I–9 shows the tooth profile of a whole depth standard rack tooth and mating gear. It has an addendum of $ha = 1m$ and dedendum $hf \geq 1.25m$. If tooth depth is shorter than whole depth it is called a "stub" tooth; and if deeper than whole depth it is a "high" depth tooth.

In the standard involute gear, pitch *p* times the number of teeth becomes the length of pitch circle:

$$d\pi = \pi m Z$$

Pitch diameter d is then:

$$d = mZ$$

Metric Module and Inch Gear Preferences: Because there is no direct equivalence between the pitches in metric and inch systems, it is not possible to make direct substitutions. Further, there are preferred modules in the metric system. As an aid in using metric gears, Table I-2 presents nearest equivalents for both systems, with the preferred sizes in bold type.

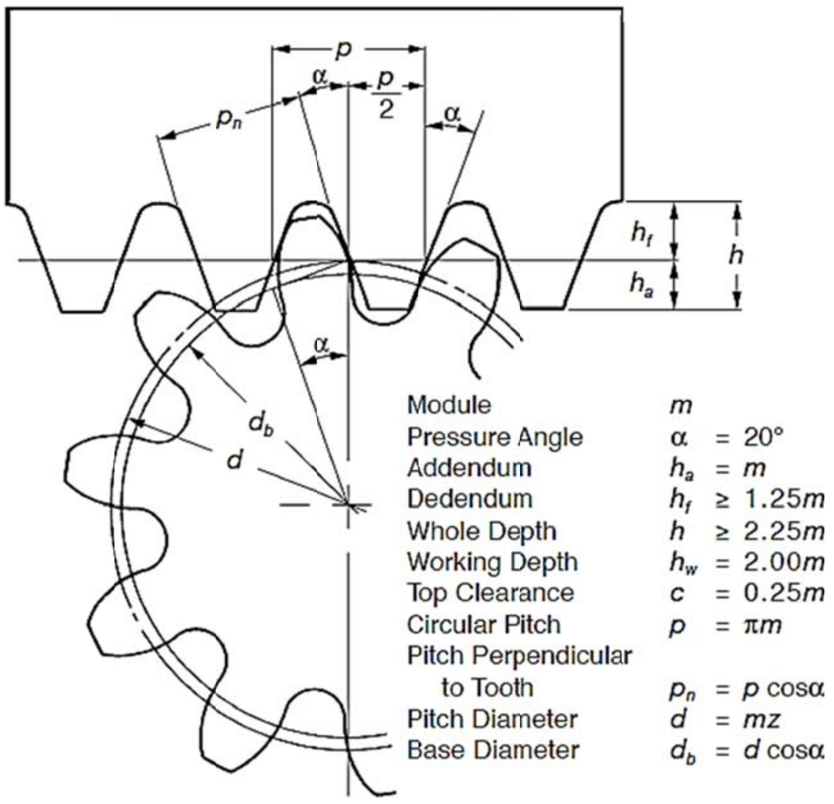

Fig. I–9 The Tooth Profile and Dimension of Standard Rack

Table I-2 Metric/American Gear Equivalents

Diametral Pitch, P	Module, m	Circular Pitch		Circular Tooth Thickness		Addendum	
		in	mm	in	mm	in	mm
203.2000	0.125	0.0155	0.393	0.0077	0.196	0.0049	0.125
200	0.12700	0.0157	0.399	0.0079	0.199	0.0050	0.127
180	0.14111	0.0175	0.443	0.0087	0.222	0.0056	0.141
169.333	0.15	0.0186	0.471	0.0093	0.236	0.0059	0.150
150	0.16933	0.0209	0.532	0.0105	0.266	0.0067	0.169
127.000	0.2	0.0247	0.628	0.0124	0.314	0.0079	0.200
125	0.20320	0.0251	0.638	0.0126	0.319	0.0080	0.203
120	0.21167	0.0262	0.665	0.0131	0.332	0.0083	0.212
101.600	0.25	0.0309	0.785	0.0155	0.393	0.0098	0.250
96	0.26458	0.0327	0.831	0.0164	0.416	0.0104	0.265
92.3636	0.275	0.0340	0.864	0.0170	0.432	0.0108	0.275
84.6667	0.3	0.0371	0.942	0.0186	0.471	0.0118	0.300
80	0.31750	0.0393	0.997	0.0196	0.499	0.0125	0.318
78.1538	0.325	0.0402	1.021	0.0201	0.511	0.0128	0.325
72.5714	0.35	0.0433	1.100	0.0216	0.550	0.0138	0.350
72	0.35278	0.0436	1.108	0.0218	0.554	0.0139	0.353
67.733	0.375	0.0464	1.178	0.0232	0.589	0.0148	0.375
64	0.39688	0.0491	1.247	0.0245	0.623	0.0156	0.397
63.500	0.4	0.0495	1.257	0.0247	0.628	0.0157	0.400
50.800	0.5	0.0618	1.571	0.0309	0.785	0.0197	0.500
50	0.50800	0.0628	1.596	0.0314	0.798	0.0200	0.508
48	0.52917	0.0655	1.662	0.0327	0.831	0.0208	0.529
44	0.57727	0.0714	1.814	0.0357	0.907	0.0227	0.577
42.333	0.6	0.0742	1.885	0.0371	0.942	0.0236	0.600
40	0.63500	0.0785	1.995	0.0393	0.997	0.0250	0.635
36.2857	0.7	0.0866	2.199	0.0433	1.100	0.0276	0.700
36	0.70556	0.0873	2.217	0.0436	1.108	0.0278	0.706
33.8667	0.75	0.0928	2.356	0.0464	1.178	0.0295	0.750
32	0.79375	0.0982	2.494	0.0491	1.247	0.0313	0.794
31.7500	0.8	0.0989	2.513	0.0495	1.257	0.0315	0.800
30	0.84667	0.1047	2.660	0.0524	1.330	0.0333	0.847
28.2222	0.9	0.1113	2.827	0.0557	1.414	0.0354	0.900
28	0.90714	0.1122	2.850	0.0561	1.425	0.0357	0.907
25.4000	1	0.1237	3.142	0.0618	1.571	0.0394	1.000
24	1.0583	0.1309	3.325	0.0654	1.662	0.0417	1.058
22	1.1545	0.1428	3.627	0.0714	1.813	0.0455	1.155
20.3200	1.25	0.1546	3.927	0.0773	1.963	0.0492	1.250
20	1.2700	0.1571	3.990	0.0785	1.995	0.0500	1.270
18	1.4111	0.1745	4.433	0.0873	2.217	0.0556	1.411
16.9333	1.5	0.1855	4.712	0.0928	2.356	0.0591	1.500
16	1.5875	0.1963	4.987	0.0982	2.494	0.0625	1.588
15	1.6933	0.2094	5.320	0.1047	2.660	0.0667	1.693
14.5143	1.75	0.2164	5.498	0.1082	2.749	0.0689	1.750
14	1.8143	0.2244	5.700	0.1122	2.850	0.0714	1.814
13	1.9538	0.2417	6.138	0.1208	3.069	0.0769	1.954
12.7000	2	0.2474	6.283	0.1237	3.142	0.0787	2.000
12	2.1167	0.2618	6.650	0.1309	3.325	0.0833	2.117
11.2889	2.25	0.2783	7.069	0.1391	3.534	0.0886	2.250
11	2.3091	0.2856	7.254	0.1428	3.627	0.0909	2.309
10.1600	2.50	0.3092	7.854	0.1546	3.927	0.0984	2.500
10	2.5400	0.3142	7.980	0.1571	3.990	0.1000	2.540
9.2364	2.75	0.3401	8.639	0.1701	4.320	0.1083	2.750
9	2.8222	0.3491	8.866	0.1745	4.433	0.1111	2.822
8.4667	3	0.3711	9.425	0.1855	4.712	0.1181	3.000
8	3.1750	0.3927	9.975	0.1963	4.987	0.1250	3.175
7.8154	3.25	0.4020	10.210	0.2010	5.105	0.1280	3.250
7.2571	3.5	0.4329	10.996	0.2164	5.498	0.1378	3.500
7	3.6286	0.4488	11.400	0.2244	5.700	0.1429	3.629
6.7733	3.75	0.4638	11.781	0.2319	5.890	0.1476	3.750
6.3500	4	0.4947	12.566	0.2474	6.283	0.1575	4.000
6	4.2333	0.5236	13.299	0.2618	6.650	0.1667	4.233
5.6444	4.5	0.5566	14.137	0.2783	7.069	0.1772	4.500
5.3474	4.75	0.5875	14.923	0.2938	7.461	0.1870	4.750
5.0800	5	0.6184	15.708	0.3092	7.854	0.1969	5.000
5	5.0800	0.6283	15.959	0.3142	7.980	0.2000	5.080
4.6182	5.5000	0.6803	17.279	0.3401	8.639	0.2165	5.500
4.2333	6	0.7421	18.850	0.3711	9.425	0.2362	6.000
4	6.3500	0.7854	19.949	0.3927	9.975	0.2500	6.350
3.9077	6.5000	0.8040	20.420	0.4020	10.210	0.2559	6.500
3.6286	7	0.8658	21.991	0.4329	10.996	0.2756	7.000
3.5000	7.2571	0.8976	22.799	0.4488	11.399	0.2857	7.257
3.1750	8	0.9895	25.133	0.4947	12.566	0.3150	8.000
3.1416	8.0851	1.0000	25.400	0.5000	12.700	0.3183	8.085
3	8.4667	1.0472	26.599	0.5236	13.299	0.3333	8.467
2.8222	9	1.1132	28.274	0.5566	14.137	0.3543	9.000
2.5400	10	1.2368	31.416	0.6184	15.708	0.3937	10.000
2.5000	10.160	1.2566	31.919	0.6283	15.959	0.4000	10.160
2.3091	11	1.3605	34.558	0.6803	17.279	0.4331	11.000
2.1167	12	1.4842	37.699	0.7421	18.850	0.4724	12.000
2	12.700	1.5708	39.898	0.7854	19.949	0.5000	12.700
1.8143	14	1.7316	43.982	0.8658	21.991	0.5512	14.000
1.5875	16	1.9790	50.265	0.9895	25.133	0.6299	16.000
1.5000	16.933	2.0944	53.198	1.0472	26.599	0.6667	16.933
1.4111	18	2.2263	56.549	1.1132	28.274	0.7087	18.000
1.2700	20	2.4737	62.832	1.2368	31.416	0.7874	20.000
1.1545	22	2.7211	69.115	1.3605	34.558	0.8661	22.000
1.0583	24	2.9684	75.398	1.4842	37.699	0.9449	24.000
1.0160	25	3.0921	78.540	1.5461	39.270	0.9843	25.000
1	25.400	3.1416	79.796	1.5708	39.898	1.0000	25.400
0.9407	27	3.3395	84.823	1.6697	42.412	1.0630	27.000
0.9071	28	3.4632	87.965	1.7316	43.982	1.1024	28.000
0.8467	30	3.7105	94.248	1.8553	47.124	1.1811	30.000
0.7938	32	3.9579	100.531	1.9790	50.265	1.2598	32.000
0.7697	33	4.0816	103.673	2.0408	51.836	1.2992	33.000
0.7500	33.867	4.1888	106.395	2.0944	53.198	1.3333	33.867
0.7056	36	4.4527	113.097	2.2263	56.549	1.4173	36.000
0.6513	39	4.8237	122.522	2.4119	61.261	1.5354	39.000
0.6350	40	4.9474	125.664	2.4737	62.832	1.5748	40.000
0.6048	42	5.1948	131.947	2.5974	65.973	1.6535	42.000
0.5644	45	5.5658	141.372	2.7829	70.686	1.7717	45.000
0.5080	50	6.1842	157.080	3.0921	78.540	1.9685	50.000
0.5000	50.800	6.2832	159.593	3.1416	79.796	2.0000	50.800

NOTE: Bold face diametral pitches and modules designate preferred values.

I.8. Gear Types And Axial Arrangements

In accordance with the orientation of axes, there are three categories of gears:
1. Parallel Axes Gears
2. Intersecting Axes Gears
3. Nonparallel and Nonintersecting Axes Gears

Spur and helical gears are the parallel axes gears. Bevel gears are the intersecting axes gears. Screw or crossed helical, worm and hypoid gears handle the third category. Table I-3 lists the gear types per axes orientation.

Also, included in Table I-3 is the theoretical efficiency range of the various gear types. These figures do not include bearing and lubricant losses. Also, they assume ideal mounting in regard to axis orientation and center distance. Inclusion of these realistic considerations will downgrade the efficiency numbers.

Table I-3 Types of Gears and Their Categories

Categories of Gears	Types of Gears	Efficiency (%)
Parallel Axes Gears	Spur Gear Spur Rack Internal Gear Helical Gear Helical Rack Double Helical Gear	98 ... 99.5
Intersecting Axes Gears	Straight Bevel Gear Spiral Bevel Gear Zerol Gear	98 ... 99
Nonparallel and Nonintersecting Axes Gears	Worm Gear Screw Gear Hypoid Gear	30 ... 90 70 ... 95 96 ... 98

I.8.1. Parallel Axes Gears

1. Spur Gear
This is a cylindrical shaped gear in which the teeth are parallel to the axis. It has the largest applications and, also, it is the easiest to manufacture. Figure I–10a

2. Spur Rack
This is a linear shaped gear which can mesh with a spur gear with any number of teeth. The spur rack is a portion of a spur gear with an infinite radius. Figure I–10b

3. Internal Gear
This is a cylindrical shaped gear but with the teeth inside the circular ring. It can mesh with a spur gear. Internal gears are often used in planetary gear systems. Figure I–10c

4. Helical Gear
This is a cylindrical shaped gear with helicoid teeth. Helical gears can bear more load than spur gears, and work more quietly. They are widely used in industry. A disadvantage is the axial thrust force the helix form causes. Figure I–10d

5. Helical Rack
This is a linear shaped gear which meshes with a helical gear. Again, it can be regarded as a portion of a helical gear with infinite radius. Figure I–10e

6. Double Helical Gear
This is a gear with both left-hand and right-hand helical teeth. The double helical form balances the inherent thrust forces. Figure I–10f

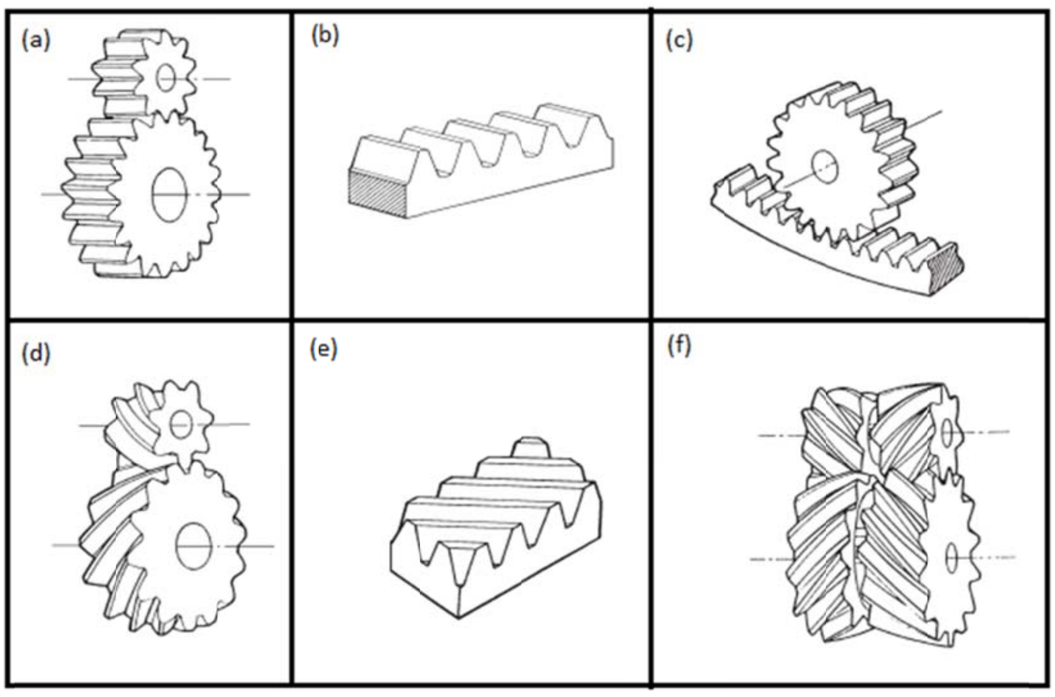

Fig. I–10 Parallel Axes Gears, (a) Spur Gear ; (b) Spur Rack ; (c) Internal Gear and Spur Gear ; (d) Helical Gear; (e) Helical Rack; (f) Double Helical Gear

I.8.2. Intersecting Axes Gears

1. Straight Bevel Gear

This is a gear in which the teeth have tapered conical elements that have the same direction as the pitch cone base line (generatrix). The straight bevel gear is both the simplest to produce and the most widely applied in the bevel gear family. Figure I–11a

2. Spiral Bevel Gear

This is a bevel gear with a helical angle of spiral teeth. It is much more complex to manufacture, but offers a higher strength and lower noise. I–11b

3. Zerol Gear

Zerol gear is a special case of spiral bevel gear. It is a spiral bevel with zero degree of spiral angle tooth advance. It has the characteristics of both the straight and spiral bevel gears. The forces acting upon the tooth are the same as for a straight bevel gear. I–11c

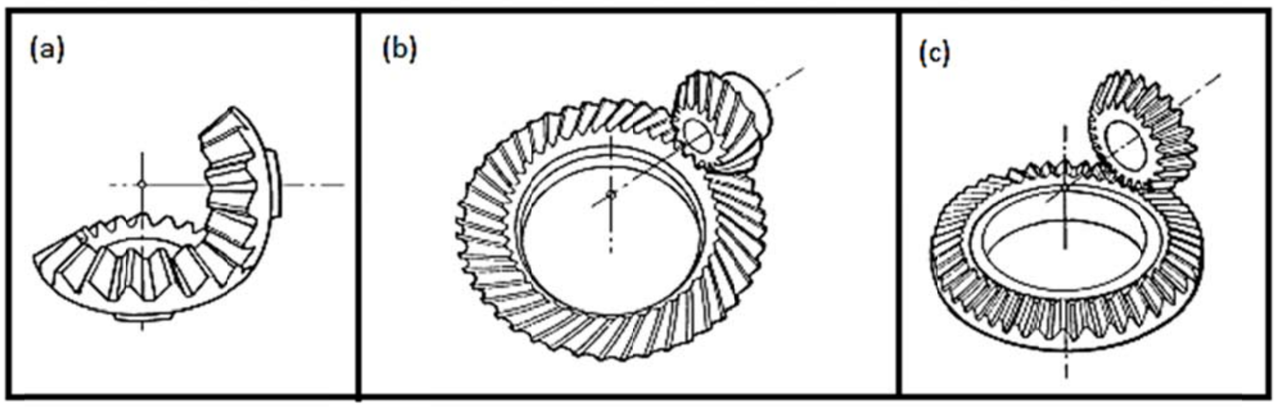

Fig. I–11 Intersecting Axes Gears, (a) Straight Bevel Gear; (b) Spiral Bevel Gear; (c) Zerol Gear

I.8.3. Nonparallel And Nonintersecting Axes Gears

1. Worm And Worm Gear

Worm set is the name for a meshed worm and worm gear. The worm resembles a screw thread; and the mating worm gear a helical gear, except that it is made to envelope the worm as seen along the worm's axis. The outstanding feature is that the worm offers a very large gear ratio in a single mesh. However, transmission efficiency is very poor due to a great amount of sliding as the worm tooth engages with its mating worm gear tooth and forces rotation by pushing and sliding. With proper choices of materials and lubrication, wear can be contained and noise is reduced. Figure I–12a

2. Screw Gear (Crossed Helical Gear)

Two helical gears of opposite helix angle will mesh if their axes are crossed. As separate gear components, they are merely conventional helical gears. Installation on crossed axes converts them to screw gears. They offer a simple means of gearing skew axes at any angle. Because they have point contact, their load carrying capacity is very limited. Figure I–12b

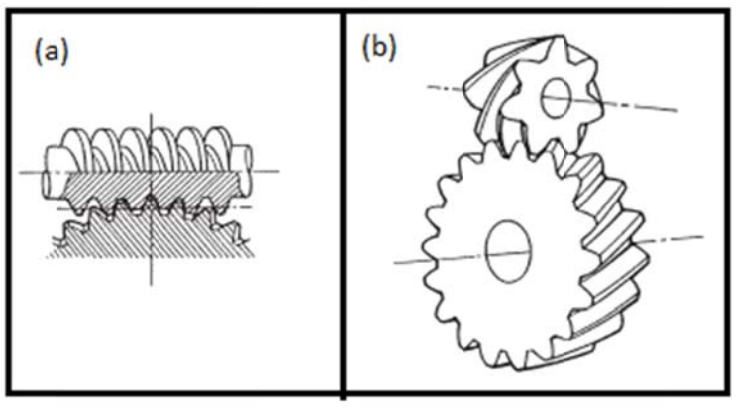

Fig. I–12 Nonparallel And Nonintersecting Axes Gears; (a) Worm Gear; (b) Screw Gear

I.8.4. Other Special Gears

1. Face Gear
This is a pseudobevel gear that is limited to 90° intersecting axes. The face gear is a circular disc with a ring of teeth cut in its side face; hence the name face gear. Tooth elements are tapered towards its center. The mate is an ordinary spur gear. It offers no advantages over the standard bevel gear, except that it can be fabricated on an ordinary shaper gear generating machine.

2. Double Enveloping Worm Gear
This worm set uses a special worm shape in that it partially envelops the worm gear as viewed in the direction of the worm gear axis. Its big advantage over the standard worm is much higher load capacity. However, the worm gear is very complicated to design and produce, and sources for manufacture are few.

3. Hypoid Gear
This is a deviation from a bevel gear that originated as a special development for the automobile industry. This permitted the drive to the rear axle to be nonintersecting, and thus allowed the auto body to be lowered. It looks very much like the spiral bevel gear. However, it is complicated to design and is the most difficult to produce on a bevel gear generator.

I.2. Spur Gear Styles

Figure I–3 shows several different styles of commercially available spur gears. When gears are large, the spoked design in Part (a) is often used to save weight. The gear teeth are machined into a relatively thin rim that is held by a set of spokes connecting to the hub. The bore of the hub is typically designed to be a close sliding fit with the shaft that carries the gear. A keyway is usually machined into the bore to allow a key to be inserted for positive transmission of torque. The first illustration does not include a keyway because this gear is sold as a stock item, and the ultimate user finishes the bore to match a given piece of equipment.

The solid hub design in Figure I–3(b) is typical of smaller spur gears. Here the finished bore with a keyway is visible. The set screw over the keyway allows the locking of the key in place after assembly.

When spur gear teeth are machined into a straight, flat bar, the assembly is called a rack, as shown in Figure I–3(c). The rack is essentially a spur gear with an infinite radius. In this form, the teeth become straight-sided, rather than the curved, involute form typical of smaller gears.

Gears with diameters between the small solid form [Part (b)] and the larger spoked form [Part (a)] are often produced with a thinned web as shown in Part (d), again to save weight. You as a designer may create special designs for gears that you implement into a mechanical device or system. One useful approach is to machine the gear teeth of small pinions directly into the surface of the shaft that carries the gear. This is very often done for the input shaft of gear reducers.

I.3. Spur Gear Geometry- Involute-Tooth Form

The most widely used spur gear tooth form is the full depth involute form. Its characteristic shape is shown in Figure I–4.

The involute is one of a class of geometric curves called *conjugate curves.* When two such gear teeth are in mesh and rotating, there is a *constant angular velocity* ratio between them: From the moment of initial contact to the moment of disengagement, the speed of the driving gear is in a constant proportion to the speed of the driven gear. The resulting action of the two gears is very smooth. If this were not the case, there would be some speeding up and slowing down during the engagement, with the resulting accelerations causing vibration, noise, and dangerous torsional oscillations in the system.

You can easily visualize an involute curve by taking a cylinder and wrapping a string around its circumference. Tie a pencil to the end of the string. Then start with the pencil tight against the cylinder, and hold the string taut. Move the pencil away from the cylinder while keeping the string taut. The curve that you will draw is an involute. Figure 1–5 is a sketch of the process.

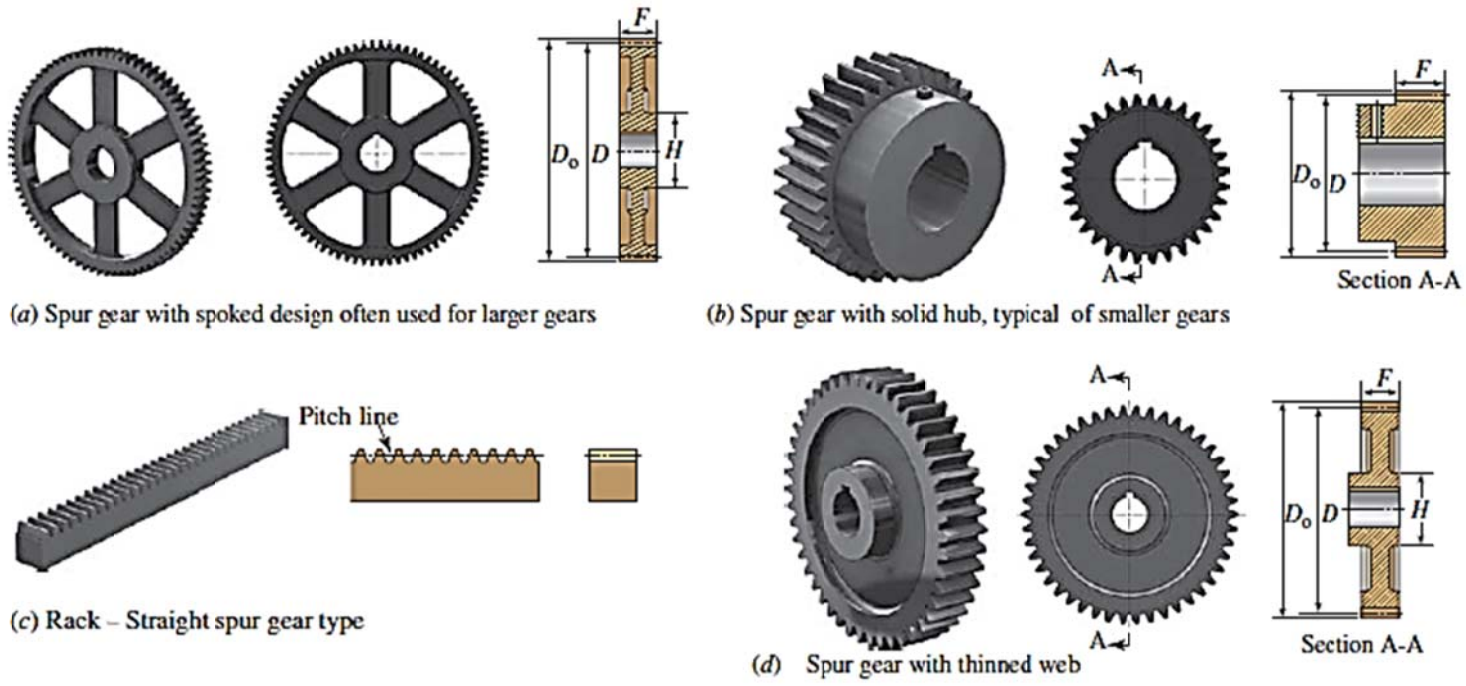

Fig. I–3 Examples of spur gears and a rack

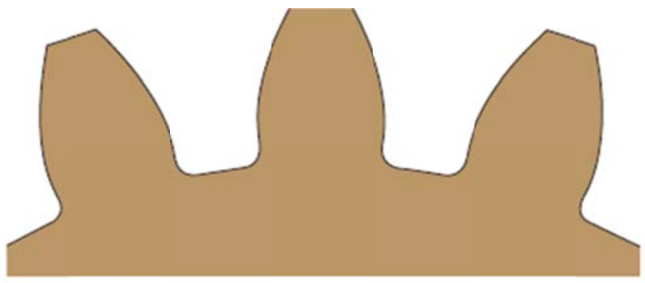

Fig. I–4 Involute-tooth form

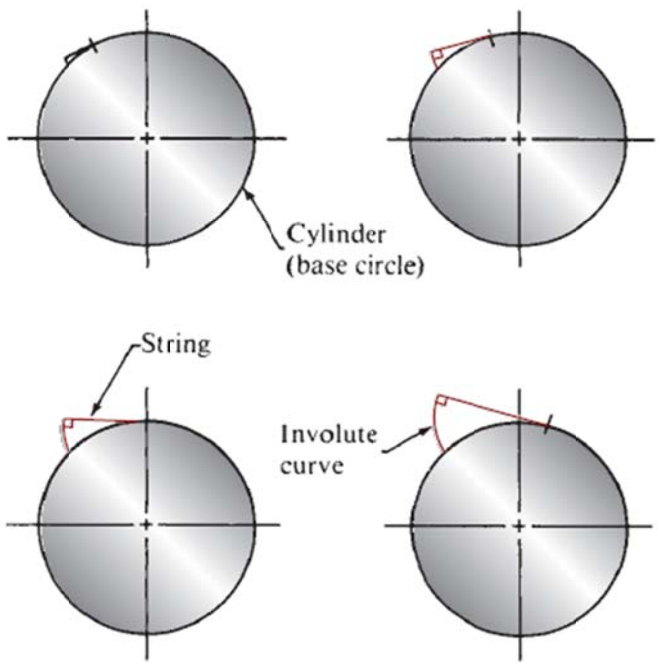

Fig. I–5 Graphical presentation of an involute curve

The circle, the end view of the cylinder, is called the *base circle*. Notice that at any position on the curve, the string represents a line tangent to the base circle and, at the same time, perpendicular to the involute. Drawing another base circle along the same centerline in such a position that the resulting involute is tangent to the first one, as shown in Figure I–6, demonstrates that at the point of contact, the two lines tangent to the base circles are coincident and will stay in the same position as the base circles rotate. This is what happens when two gear teeth are in mesh.

It is a fundamental principle of *kinematics,* the study of motion, that if the line drawn perpendicular to the surfaces of two rotating bodies at their point of contact always crosses the centerline between the two bodies at the same place, the angular velocity ratio of the two bodies will be constant. This is a statement of the *law of gearing.* As demonstrated here, the gear teeth made in the involute-tooth form obey the law. Of course, only the part of the gear tooth that actually comes into contact with the mating tooth needs to be in the involute form.

Figure I–7 shows the gear tooth involute profile. The tooth profile is the curve forming the side of the tooth and is bounded by the major diameter and minor diameters of the gear. The active profile is the portion of the gear tooth involute curve that makes contact with the mating gear. The *start of the active profile* (SAP) corresponds to the lowest point on the active profile. The *true involute form* (TIF) diameter is the diameter above which the tooth profile is a true involute, but the mating gear teeth do not contact. The tip chamfer is a tooth modification used to eliminate the sharp edge of the gear tooth flank and the major diameter of the gear.

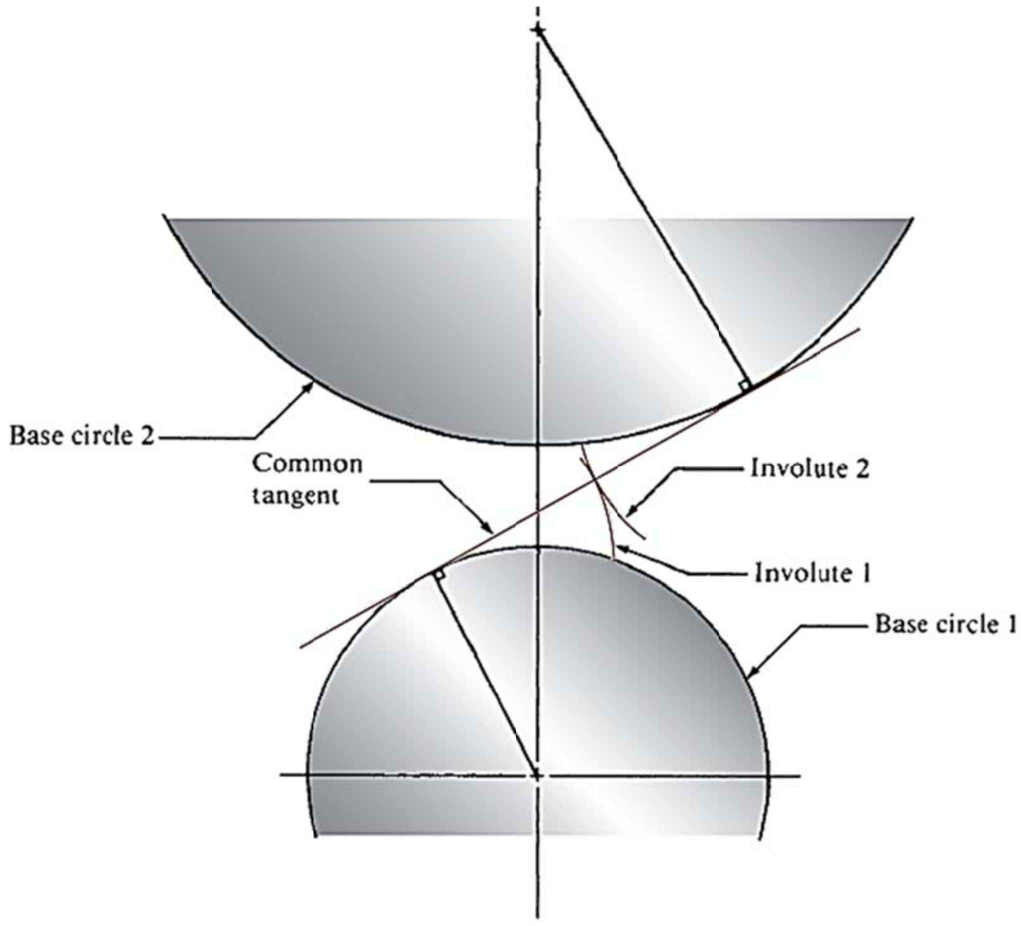

Fig. I–6 Mating involutes

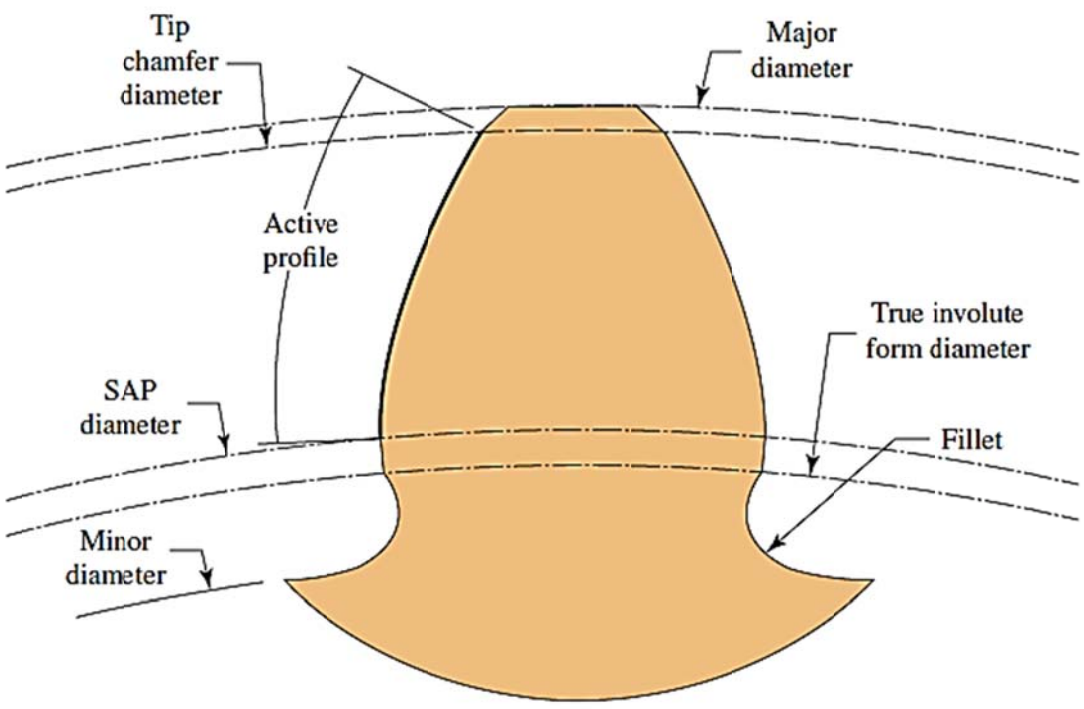

Fig. I–7 Gear tooth profile

I.4. Spur Gear Nomenclature and Gear-Tooth Features

This section describes several features of individual spur gear teeth, complete gears, and the basic geometry of two mating gears. Terms and symbols used here conform mostly to American Gear Manufacturers Association (AGMA) standards. Because there is variation among the several applicable standards, the primary reference is AGMA 2001-D04 *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*. This standard is the basis for analytical design methods for spur gears and helical gears, respectively. Where appropriate, the terms and symbols used by other AGMA standards and international standards such as ISO, DIN (Germany), and JIS (Japan) are noted. Both the conventional U.S. system of units, called the *Diametral Pitch System*, and the SI metric system, called the *Metric Module System*, are discussed. Reference is made to several figures and tables that depict the geometry of interest in the design of gear pairs:

1. Figure I–1 shows two mating spur gears, indicating the dimensions related to diameters and center distance.
2. Figure I–8 shows details of spur gear teeth with the many terms used to denote specific parts of the teeth and their relationship with the pitch diameter. These terms are defined later in this section.
3. Figure I–9 shows two gears in mesh with several important diameters, center distance, and other features.

4. Figure I–10 shows how spur gear teeth engage as the gears rotate. Gear 1 rotates clockwise and drives gear 2 that rotates counterclockwise. The teeth on gear 1, labeled *A*1, *B*1, *C*1, and *D*1, contact the teeth on gear 2, labeled *A*2, *B*2, *C*2, and *D*2 respectively. The contact between any two teeth remains along the line of action, until the teeth are no longer engaged.
5. Figures I–11 and I–12 show various sizes of gear teeth in both the diametral pitch and metric module systems. Both figures are full size, enabling you to compare physical gears to the drawings to gain an appreciation of gear tooth sizes.
6. Table I–1 is a composite reference tool for identifying the names, symbols, definitions, units, and formulas related to the several features of gear teeth and mating gears.

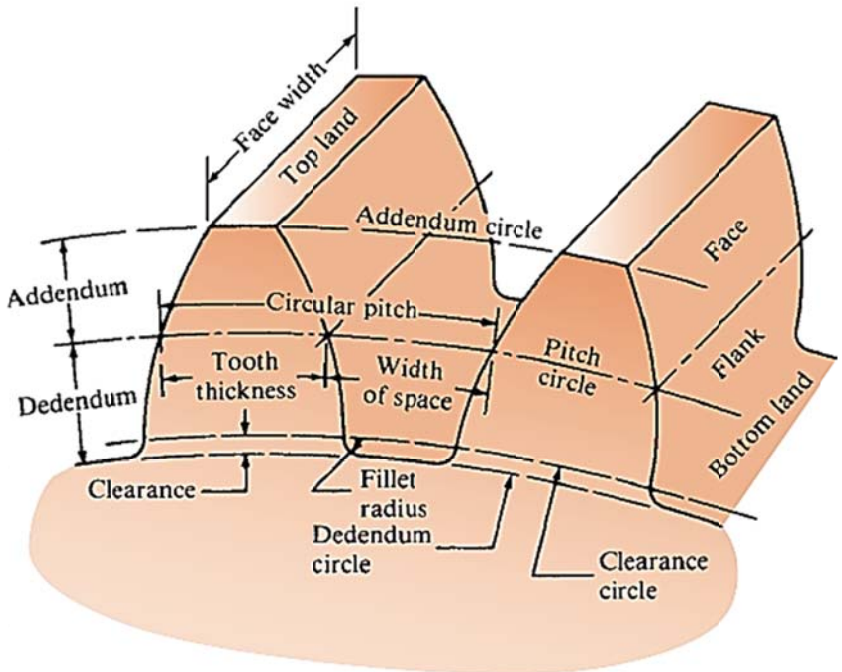

Fig. I–8 Spur gear teeth features

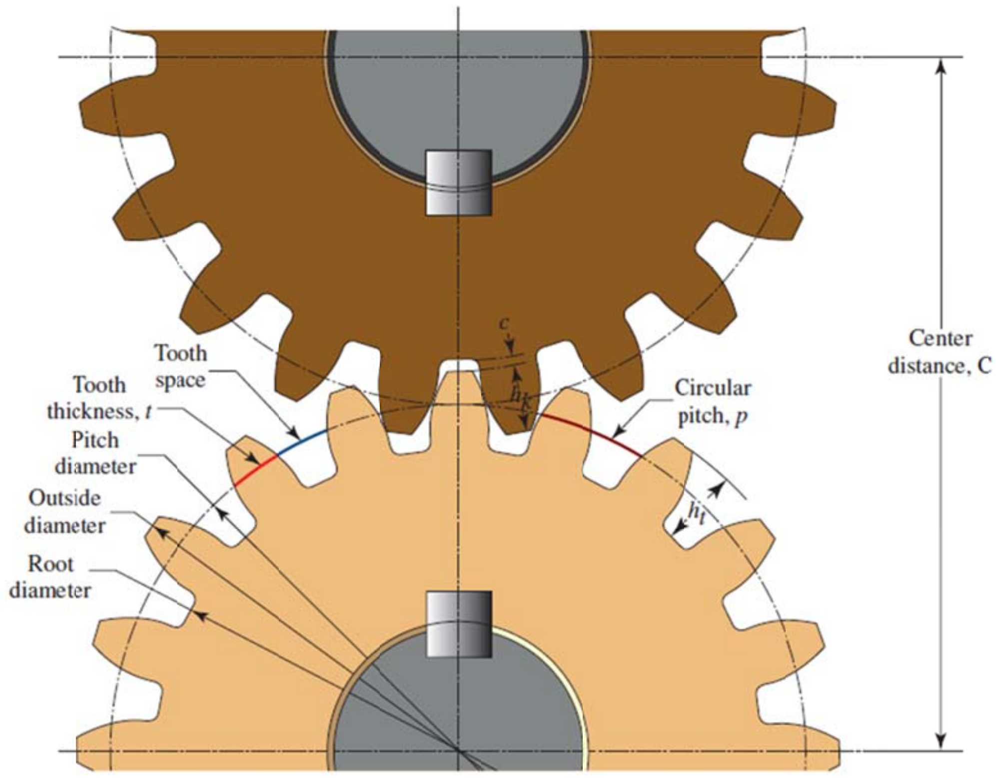

Fig. I–9 Details of two meshing spur gears showing several important geometric features.

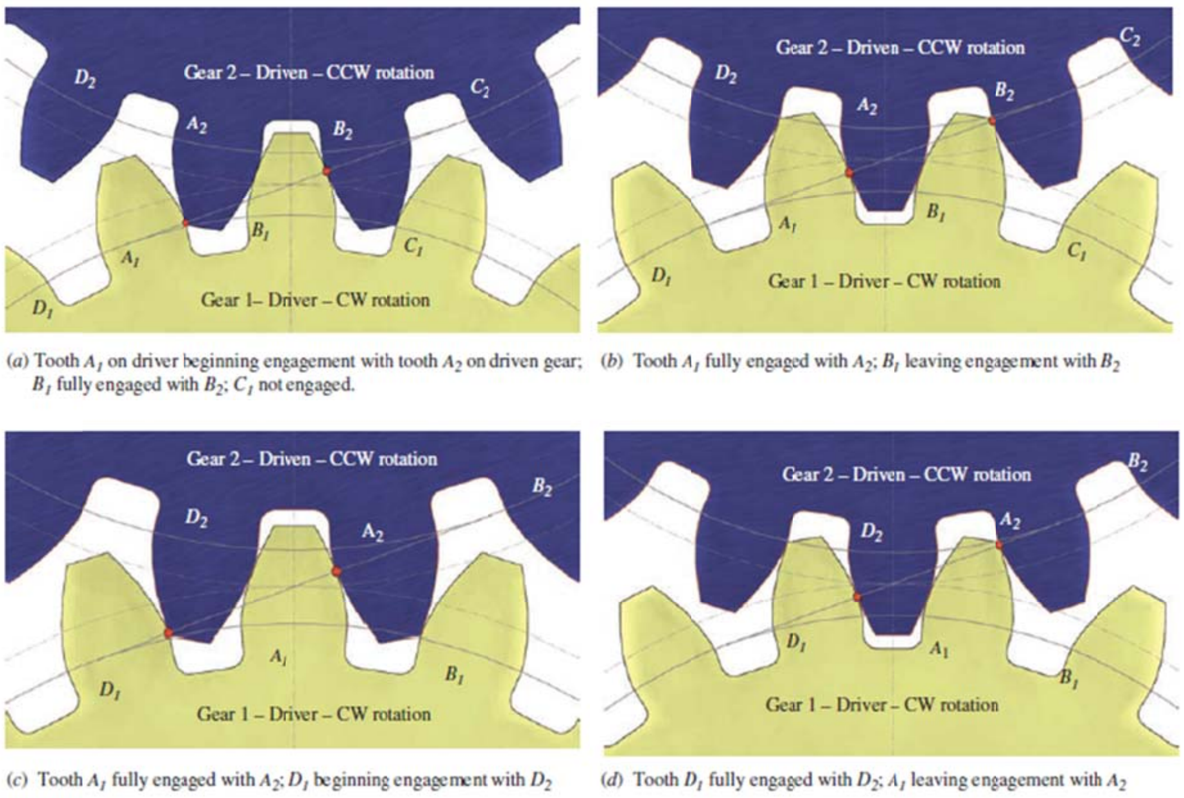

Fig. I–10 Cycle of engagement of gear teeth

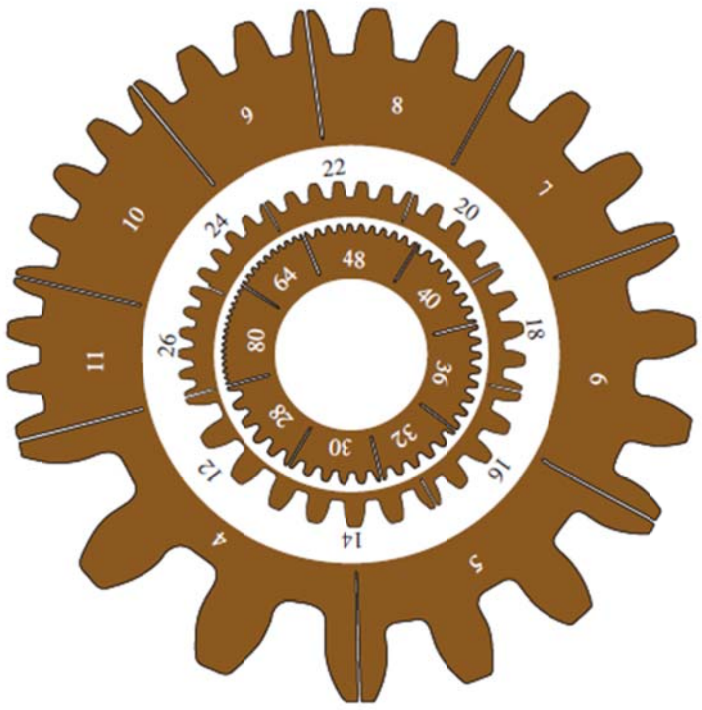

Fig. I–11 Gear-tooth size as a function of diametral pitch—actual size

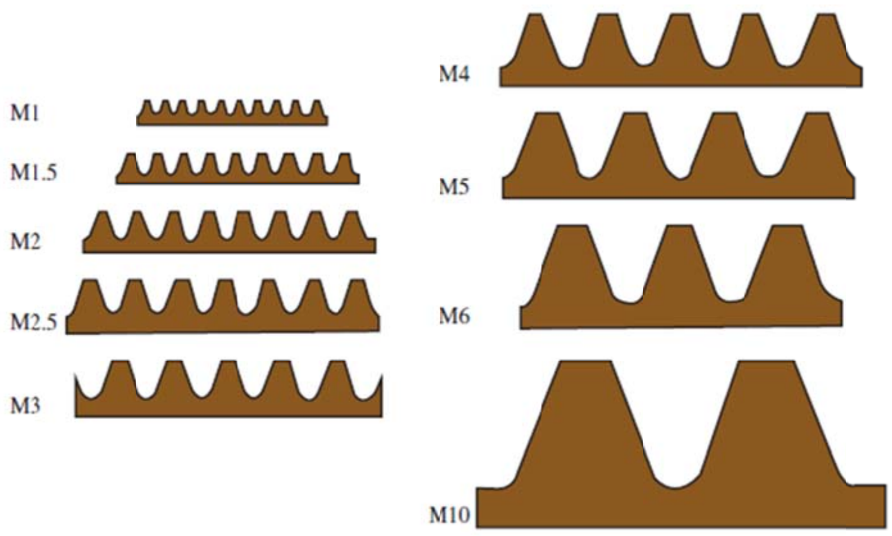

Fig. I–12 Selected standard metric modules in rack form–actual size

I.5. A note about accuracy: Gears and gear trains are precision mechanical devices with tolerances on critical dimensions typically in the range of a few ten thousandths of an inch (0.0001 in or about 0.0025 mm). Therefore, it is expected that such dimensions be reported to at least the nearest ten thousandth of an inch (four decimal places) or the nearest 0.001 mm. Some applications require even more precision.

- ***Pinion* and *Gear*.** For two gears in mesh, the smaller gear is called the *pinion* and the larger is called, simply, the *gear.*
- **Number of Teeth, *N*.** It is essential that there are an integer number of teeth in any gear. This text uses the symbol *N* for the number of teeth, with *NP* for the pinion and N_G for the gear. These subscripts are applied to other gear features as well. Another commonly used symbol for the number of teeth is *z*, with similar subscripts or simply called z_1 and z_2.
- ***Pitch*.** Refer to Figures I–8 and I–9. The *pitch* of a gear, in general, is defined as follows:

The pitch of a gear is the arc distance from a point on a tooth at the pitch circle to the corresponding point on the next adjacent tooth, measured along the pitch circle.
The pitch circle is defined next. It is important to note that the pitch of both mating gears must be identical to ensure the smooth engagement of the teeth as the gears rotate. Standard pitches are defined in three different systems, described later in this section.

- ***Pitch Circle* and *Pitch Diameter*.** When two gears are in mesh, they behave as if two smooth rollers are rolling on each other without slipping. The surface of each roller defines the *pitch circle* and its diameter is called the *pitch diameter.* The *pitch diameter*, called *D* in this lesson, is used as the characteristic size of the gear for calculations of speeds. Note that the pitch diameter for a gear is a theoretical concept and cannot be measured directly. It falls within the gear teeth and is dependent on which standard system for pitch is specified for a particular gear pair. The commonly used units for *D* are inches (in) for the U.S. system and millimeters (mm) for the SI metric system. Figures I–1, I–9, and I–14 show the pitch diameters on meshing gears.

- **Circular Pitch, p.** The pitch corresponding exactly to the basic definition of pitch given above is called the *circular pitch, p*. Some large gears that are made by casting are made to standard sizes of circular pitch such as those listed in Table I–2. They represent a very small portion of gears in common use. The formula for *p* comes from dividing the circumference of the pitch circle of the gear into *N* parts. That is,

- **Circular Pitch**

$$p = \pi D/N$$

  - ***Diametral Pitch*, Pd.** The most common pitch system in use in the United States at this time is *diametral pitch system.* We use the symbol, P_d, to denote *diametral pitch.* Note that some references use the term D_P. The definition of P_d is stated here for either the pinion or the gear and both must be identical.
  - ***Diametral Pitch***

$$P_d = N_P/D_P = N_G/D_G$$

Analysis of units shows that *Pd* has the unit of in^{-1}, but the unit is rarely reported. It is necessary to not confuse the terms *diametral pitch*, *Pd*, and *pitch diameter, D*. Note that designers often refer to gears in this system as, for example, 8-pitch for $P_d = 8$ and 20-pitch for $P_d = 20$.
In this book, we use only those values of *Pd* listed in Table I–3 because they are the most readily available as stock gears and most gear manufacturers have tooling for these sizes. Smaller pitches have larger teeth; larger pitches have smaller teeth. Note that pitches under 20 are called *coarse*, while those 20 and higher are called *fine*. Refer to Figure I–12 that shows actual sizes of teeth with certain diametral pitches. ***Note that not all listed values of* Pd *are readily available and those such as 7, 9, 11, 22, and 26 should not be specified.***

Table I-1 Gear and Tooth Features, Diameters, Center Distance for a Gear Pair

				Formulas			
					U.S. Full-depth involute system		
Number of teeth and Pitches	Symbol	Definition	Typical unit	General formula	Coarse pitch $P_d < 20$ (in)	Fine pitch $P_d \geq 20$ (in)	Metric module system (mm)
Number of teeth	N	Integer count of teeth on a gear					
Circular pitch	p	Arc distance between corresponding points on adjacent teeth	in or mm	$p = \pi D/N$	$p = \pi/P_d$		$p = \pi m$
Diametral pitch	P_d	Number of teeth per inch of pitch diameter	in^{-1}	$P_d = N/D$			
Module	m	Pitch diameter divided by number of teeth	mm	$m = D/N$			$m = 25.4/P_d$
Diameters							
Pitch diameter	D	Kinematic characteristic diameter for a gear; Diameter of the pitch circle	in or mm		$D = N/P_d$		$D = mN$
Outside diameter	D_o	Diameter to the outside surface of the gear teeth	in or mm		$D_o = (N + 2)/P_d$		$D_o = m(N + 2)$
Root diameter	D_R	Diameter to the root circle of the gear at the base of the teeth	in or mm	$D_R = D - 2b$			
Gear Tooth Features							
Addendum	a	Radial distance from pitch circle to outside of tooth	in or mm		$a = 1.00/P_d$		$a = 1.00m$
Dedendum	b	Radial distance from pitch circle to bottom of tooth space	in or mm		$b = 1.25/P_d$	$b = 1.20/P_d + 0.002$	$b = 1.25m^1$
Clearance	c	Radial distance from top of fully engaged tooth of mating gear to bottom of tooth space	in or mm		$c = 0.25/P_d$	$c = 0.20/P_d + 0.002$	$c = 0.25m^1$
Whole depth	h_t	Radial distance from top of a tooth to bottom of tooth space	in or mm	$h_t = a + b$	$h_t = 2.25/P_d$	$h_t = 2.20/P_d + 0.002$	$h_t = 2.25m^1$
Working depth	h_k	Radial distance a gear tooth projects into tooth space of mating gear	in or mm	$h_k = a + a = 2a$	$h_k = 2.00/P_d$	$h_k = 2.00/P_d$	$h_k = 2.00m^1$
Tooth thickness	t	Theoretical arc distance equal to 1/2 of circular pitch	in or mm	$t = p/2$	$t = \pi/[2(P_d)]$		$t = \pi m/2$
Face width	F	Width of tooth parallel to axis of gear	in or mm	*Design decision*	Approximately $12/P_d$		
Pressure angle	ϕ	Angle between the tangent to the pitch circle and the perpendicular to the gear tooth surface	degrees	*Design decision*	Most common value = 20° Others: 14 1/2°, 25°		
Center Distance	C	Distance from between centerlines of mating gears	in or mm	$C = (D_P + D_G)/2$	$C = (N_P + N_G)/2P_d$		$C = m(N_P + N_G)/2$

Note: [1]Factors in formula for dedendum may vary in metric module system to obtain custom clearance.

I.6. Number of Teeth Related to Pitch Diameter

The relationship between the pitch diameter, diametral pitch, and the number of teeth is given by Equation I–3. As the number of teeth increases for a given diametral pitch, the pitch diameter increases. So as the pitch diameter increases, the base circle diameter increases. A larger base circle will decrease the curvature of the involute curve of the gear tooth. Figure I–13 shows as the number of teeth increases for a given diametral pitch, the radius of the involute curve of the gear tooth will decrease. Inversely, as the number of teeth decrease the radius of the involute curve of the gear tooth will increase. The teeth in this figure are all drawn with a diametral pitch of 5. The gear rack can be thought of as a gear with an infinitely large diameter, where the involute curve of the tooth profile becomes a straight line.

Metric Module, *m*. The basic definition of the metric module, *m*, is given here for both the pinion and the gear and they must be identical.

* **Metric Module**

  * $m = D_p/N_p = D_G/N_G$

  The unit of mm is typically used. Note that smaller values of *m* denote smaller teeth and vice versa. Some references and vendors' tables use the symbol, M, for module and write M5 for $m = 5$, for example. Figure I–12 shows actual sizes of nine standard modules, illustrated as the teeth of *racks*, straight gears with infinite diameters. Many more modules are available and Table I–4 lists a total of 19 values that cover typical applications featured in this book.

  Relation between *Pd* and *m*. With globally integrated design and marketing of products and systems, it is likely that the need to convert from one system to another will be needed. Note that the definition of *Pd* is fundamentally the inverse of the definition of *m*. That is,

  $$m = 1/Pd \text{ or } Pd = 1/m$$

  However, because different units are employed for each term, a conversion factor of 25.4 mm/in is required, resulting in useful forms of the relationship as

* **Relation between Module and Diametral Pitch**

  $$m = 25.4/Pd \text{ or } Pd = 25.4/m$$

  Table I–4 uses this relationship to compute the equivalent diametral pitch, *Pd*, for given standard metric modules, *m*. Note that the conversions do not deliver standard values of *Pd* such as those listed in Table I–3. Therefore, we list the nearest standard *Pd* value in Table I–4 as an aid to designers who are considering converting a design from one system to the other. We can say, for example, that a module of $m = 1.25$ is a closely similar size to $Pd = 20$. Of course, the design for either system must be completed independently.

  Gear Tooth Features. Table I–1 includes the definitions of several other features of individual teeth that designers must be familiar with. Refer to Figures I–8 and I–9 to visualize those features.

  Face Width, *F*. Face width is the width of the gear parallel to the axis of the gear. It is defined by the designer as one of the required *design decisions*. For now, we can state that a nominal value for face width is approximately $F \sim 12/P_d$, but a wide range is permitted.

* **Center Distance,** C. One of the most critical dimensions for a gear pair is the *center distance*, defined as the linear distance from the centerline of the pinion to the centerline of the gear as shown in Figure I–1. The theoretical value is best represented as the sum of the pitch radii of the pinion and the gear. That is,

  Pitch radii: $R_P = D_P/2$ and $R_G = D_G/2$

  Center distance: $C = R_P + R_G = D_P/2 + D_G/2$

  $$C = (D_P + D_G)/2$$

  Other useful equations for *C*, recommended for use in this book, are developed here.

Table I-2 Standard Circular Pitches (in)

10.0	7.5	5.0
9.5	7.0	4.5
9.0	6.5	4.0
8.5	6.0	3.5
8.0	5.5	

Table I-3 Standard Diametral Pitches (teeth/in)

Coarse pitch ($P_d < 20$)				Fine pitch ($P_d \geq 20$)	
1	2	5	12	20	72
1.25	2.5	6	14	24	80
1.5	3	8	16	32	96
1.75	4	10	18	48	120
				64	

(a) $N = 11$ (b) $N = 20$ (c) $N = 40$ (d) Rack

Fig. I–13 Involute curve shape for varying numbers of teeth for a diametral pitch of 5

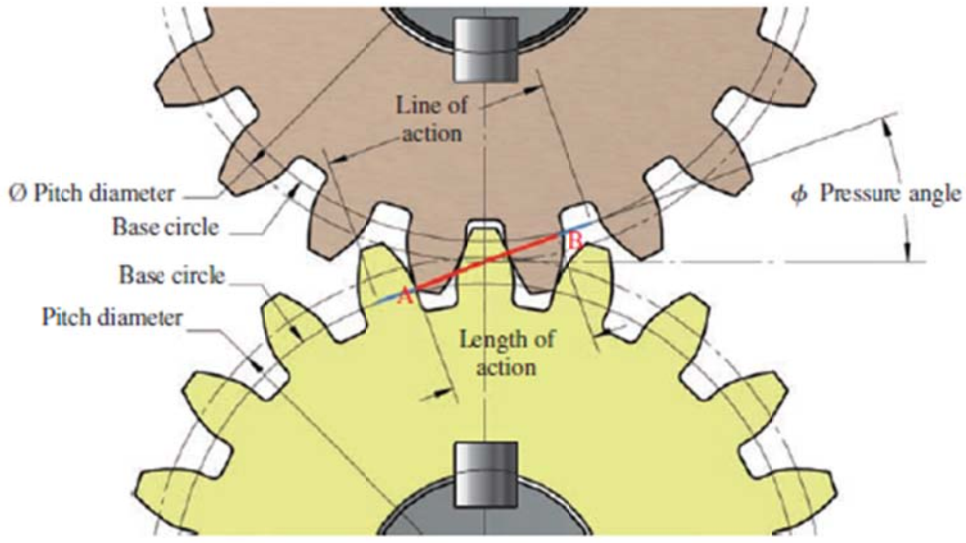

Fig. I–14 Two spur gears in mesh showing the pressure angle, line of action, base circles, pitch diameters, and other features

Table I-4 Standard modules

Module (mm)	Equivalent P_d	Closest standard P_d (teeth/in)
0.3	84.667	80
0.4	63.500	64
0.5	50.800	48
0.8	31.750	32
1	25.400	24
1.25	20.320	20
1.5	16.933	16
2	12.700	12
2.5	10.160	10
3	8.466	8
4	6.350	6
5	5.080	5
6	4.233	4
8	3.175	3
10	2.540	2.5
12	2.117	2
16	1.587	1.5
20	1.270	1.25
25	1.016	1

- **Center Distance in terms of *NG*, *NP*, and *Pd***

$$D_P = N_P/P_d \text{ and } D_G = N_G/P_d$$
$$C = (D_P + D_G)/2 = (N_P/P_d + N_G/P_d)/2$$
$$C = (N_P + N_G)/2P_d$$

- **Center Distance in terms of *NG*, *NP*, and m**

$$DP = mN_P \text{ and } D_G = mN_G$$
$$C = (D_P + D_G)/2 = (mN_P + mN_G)/2$$
$$C = m(N_P + N_G)/2$$

An important advantage of the final forms of the center distance is that all numbers on the right side are typically integers or exact fractional dimensions such as 1.25, 1.5, or 2.5. Therefore, the highest level of accuracy is obtained from using those forms. Conversely, some values for pitch diameters are irrational numbers. For example, a 12-pitch gear with 65 teeth has a pitch diameter of

$$D = N/P_d = 65/12 = 5.416666 \text{ in and}$$
$$R = D/2 = 2.708333\ldots\text{in}$$

Depending on rounding, an inaccurate calculation for center distance could result from using these values.

Comments on Gear Tooth Features. The definitions and formulas given in Table I–1 yield the theoretical values and it is typical for gear designers and manufacturers to modify some features to produce preferred performance characteristics. It is important to realize that some of these practices change the fundamental geometry of the gears and they may result in weaker tooth shapes, vibration during operation, and/or increased noise. Some examples are given below:

1. *Addendum modification:* The upper part of the flank of a gear tooth is the first to engage its mating tooth and it penetrates most deeply into the tooth space of the mating gear. Some applications benefit from relieving the true surface of the tooth or from shortening the addendum to promote smooth engagement or to avoid damage.

2. *Dedendum modification:* Some designers prefer greater clearance between the bottom of the tooth space and the fully engaged mating tooth to facilitate lubrication. Using a longer dedendum dimension provides this capability. Plastic gears may also employ dedendum modification to accommodate thermal expansion and/or swelling because of moisture absorption.

3. *Center distance modification:* Some combinations of gear tooth geometry result in very small clearances between teeth and possibly interference between the top of the engaging tooth and the lower part of the flank of the mating tooth. These conditions should be avoided by proper design decisions to avoid the interference.
However, some designers remove the interference by expanding the center distance slightly from its theoretical dimension.

4. *Tooth thickness modification:* A tooth thickness defined exactly as 1/2 of the circular pitch will result in a tight fit of a tooth in the tooth space of the mating gear, possibly causing binding or precluding entry of lubrication at the contact point on the teeth. Using a smaller tooth thickness provides space for lubrication and facilitates assembly. The space created is called *backlash* and it is discussed next.

- ***Backlash:*** To provide backlash, the cutter generating the gear teeth can be fed more deeply into the gear blank than the theoretical value on either or both of the mating gears. Alternatively, backlash can be created by adjusting the center distance to a larger value than the theoretical value. The magnitude of backlash depends on the desired precision of the gear pair and on the size and the pitch of the gears. It is actually a design decision, balancing cost of production with desired performance. The American Gear Manufacturers Association (AGMA) provides recommendations for backlash in their standards. Table I–5 lists examples of recommended ranges for several values of pitch.

Table I-5 Recommended Minimum Backlash for Coarse Pitch Gears

A. Diametral pitch system (backlash in inches)

	Center distance, *C* (in)				
P_d	2	4	8	16	32
18	0.005	0.006			
12	0.006	0.007	0.009		
8	0.007	0.008	0.010	0.014	
5		0.010	0.012	0.016	
3		0.014	0.016	0.020	0.028
2			0.021	0.025	0.033
1.25				0.034	0.042

B. Metric module system (backlash in millimeters)

	Center distance, *C* (mm)				
Module, *m*	50	100	200	400	800
1.5	0.13	0.16			
2	0.14	0.17	0.22		
3	0.18	0.20	0.25	0.35	
5		0.26	0.31	0.41	
8		0.35	0.40	0.50	0.70
12			0.52	0.62	0.82
18				0.80	1.00

1.7. Pressure Angle

The* pressure angle *is the angle between the tangent to the pitch circles and the line drawn normal (perpendicular) to the surface of the gear tooth (see Figure I–14).

The normal line is sometimes referred to as the *line of action*. When two gear teeth are in mesh and are transmitting power, the force transferred from the driver to the driven gear tooth acts in a direction along the line of action. The *length of action* is the distance AB along the line of action. This length represents the path of the point of contact during the meshing of the two gears.

Also, the actual shape of the gear tooth depends on the pressure angle, as illustrated in Figure I–15. The teeth in this figure were drawn according to the proportions for a 20-tooth, 5-pitch gear having a pitch diameter of 4.000 in. All three teeth have the same tooth thickness because, as stated in Table I–1 the thickness at the pitch line depends only on the pitch. The difference between the three teeth shown is due to the different pressure angles because the pressure angle determines the size of the base circle. Remember that the base circle is the circle from which the involute is generated. The line of action is always tangent to the base circle. Therefore, the size of the base circle can be found from

- **Base Circle Diameter**

$$D_b = D \cos \phi$$

As the pressure angle increases, the thickness at the bottom of the tooth increases. Therefore the larger the pressure angle, the stronger the tooth and the higher the load carrying capacity of the tooth. A higher pressure angle may not run as smoothly or quietly as the smaller pressure angle. As the pressure angle decreases, the top land of the tooth will increase. This can create interference problems when a small number of pinion teeth are required.

Standard values of the pressure angle are established by gear manufacturers, and the pressure angles of two gears in mesh must be the same. Current standard pressure angles are (14 1/2)°, 20°, and 25° as illustrated in Figure I–15. Actually, the (14 1/2)° tooth form is considered obsolete. Although it is still available, it should be avoided for new designs. The 20° tooth form is the most readily available at this time. The advantages and disadvantages of the different values of pressure angle relate to the strength of the teeth, the occurrence of interference, and the magnitude of forces exerted on the shaft.

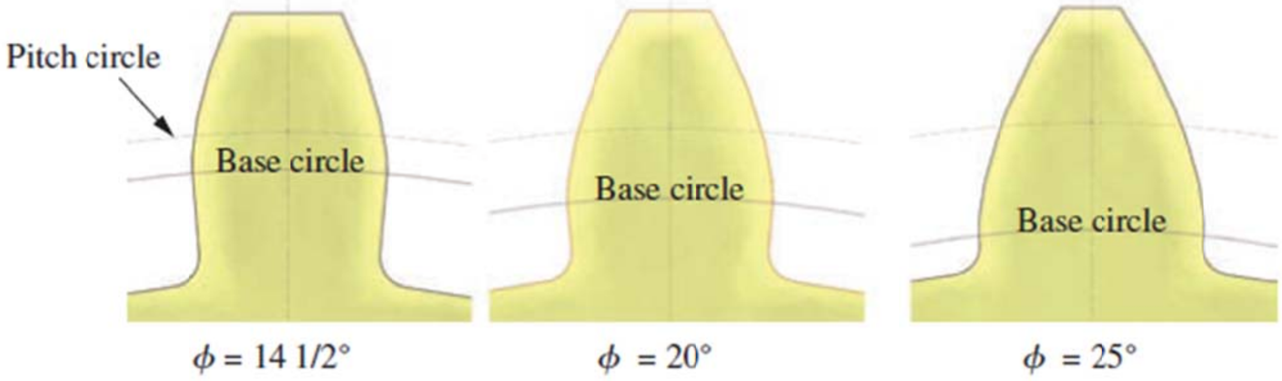

Fig. I–15 Illustration of how the shape of gear teeth change as the pressure angle, (phi), changes