Solution série de TD N°1.

Exo1: – un signal periodique de periode T_0 celà veut dire :

$s(t+T_0) = s(t)$,

* $s_1(t) = 3\cos 12\pi t = 3\cos 2\pi \times 6 \times t$ (périodique de periode $T_0 = \frac{1}{6}s$)
* $s_2(t) = 4\exp(4t)u(t)$ (non périodique)
* $s_3(t) = 14\exp(-6t)u(t)$ (non périodique)
* $s_4(t) = 8\exp(-2|t|)$ (non périodique)
* $s_5(t) = \sin 3\pi t + 3\cos 4\pi t = \underbrace{\sin 2\pi \times \frac{3}{2} t}_{\text{periode } T_1 = \frac{2}{3}} + \underbrace{3\cos 2\pi \times 2 \times t}_{\text{periode } \frac{1}{2} = T_2}$.

$s_5(t)$: periodique si $\exists$ n_1 et n_2 entiers

tels que $n_1 \times T_1 = n_2 T_2$.

donc $n_1 \times \frac{2}{3} = n_2 \times \frac{1}{2}$ $\left| \begin{array}{l} n_1 = 3 \text{ et } n_2 = 4 \\ \text{verifient la relation} \end{array} \right)$.

donc $s_5(t)$ est periodique de periode $T_0 = 3T_1 = 4T_2 = 2s$.

* $s_6(t) = 10\exp(-4t).\cos 10\pi t$ (non périodique)
* $s_7(t) = \sin 2t + \sin 2\pi t = \underbrace{\sin 2\pi(\frac{1}{\pi})t}_{\text{periode } T_1 = \pi} + \underbrace{\sin 2\pi \times 1 t}_{\text{periode } T_2 = 1}$.

$\nexists$ n_1 et n_2 tel que $n_1 T_1 = n_2 T_2$.

donc $s_7(t)$ est non periodique.

* $s_8(t) = 3|\cos 2\pi f_0 t|$ (périodique) de période $\frac{1}{2f_0} = T$

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* les signaux à energie finie sont :

$s_3(t)$, $s_4(t)$

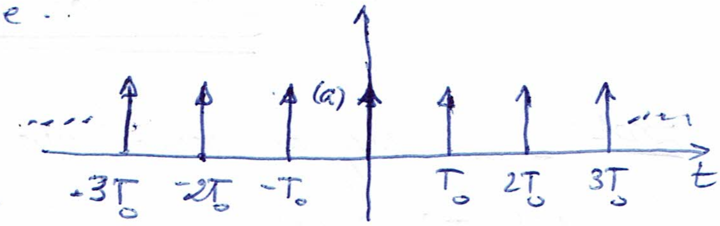
* les signaux à puissance finie sont

$s_1(t)$, $s_5(t)$, $s_7(t)$ et $s_8(t)$,

pour un signal periodique $P = \frac{1}{T_0}\int_{(T_0)} s^2(t)\,dt$.

<u>Exo2</u> : série de Fourier Complexe.

1) $s_1(t) = a \sum_{n=-\infty}^{+\infty} \delta(t - nT_0)$



$$s(t) = \sum_{n=-\infty}^{+\infty} C_n \cdot e^{+j2\pi n f_0 t} \quad \text{avec } f_0 = \frac{1}{T_0}.$$

$$C_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} s(t) e^{-j2\pi n f_0 t} dt$$

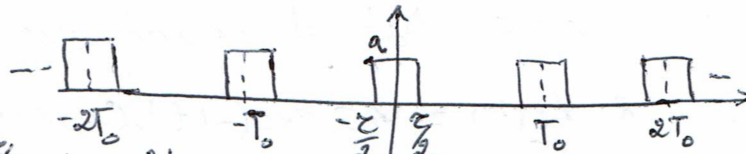
$$C_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} a.\delta(t).e^{-j2\pi n f_0 t} dt = \frac{a}{T_0} \cdot e^{-j2\pi n f_0 t}\Big|_{t=0} = \frac{a}{T_0}.$$

par application de la propriété de $\delta(t)$.

<u>donc</u> : $s_1(t) = \frac{a}{T_0} \sum_{n=-\infty}^{+\infty} e^{+j2\pi n f_0 t}$

$$TF\{s_1(t)\} = \sum_{n=-\infty}^{+\infty} C_n \delta(f - nf_0) = \frac{a}{T_0} \sum_{n=-\infty}^{+\infty} \delta(f - nf_0)$$

2) $s_2(t) = a \sum_{n=-\infty}^{+\infty} rect_\tau(t - nT_0)$



$$C_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} s_2(t) e^{-j2\pi n f_0 t} dt = \frac{1}{T_0} \int_{-\tau/2}^{\tau/2} a.e^{-j2\pi n f_0 t} dt = \frac{a}{T_0} . \tau \operatorname{sinc} \pi n f_0 \tau.$$

$$\text{donc} \quad s_2(t) = \sum_{n=-\infty}^{+\infty} \frac{a}{T_0} . \tau \operatorname{sinc} \pi n f_0 \tau . e^{j2\pi n f_0 t}.$$

$$TF\{s_2(t)\} = \sum_{n=-\infty}^{+\infty} \frac{a}{T_0} \tau \operatorname{sinc} \pi n f_0 \tau . \delta(f - nf_0).$$

<u>Remarque</u> : le spectre d'un signal periodique est discret.

<u>Exo3</u> :

$$s(t) = \begin{cases} 2 \exp(-2t) & t \geq 0 \\ 0 & \text{ailleurs} \end{cases}$$

la Transformée de Fourier est donnée par :

$$S(f) = \int_{-\infty}^{+\infty} s(t) e^{-j2\pi f t} dt = \int_0^{+\infty} 2 e^{-2t} . e^{-j2\pi f t} dt$$

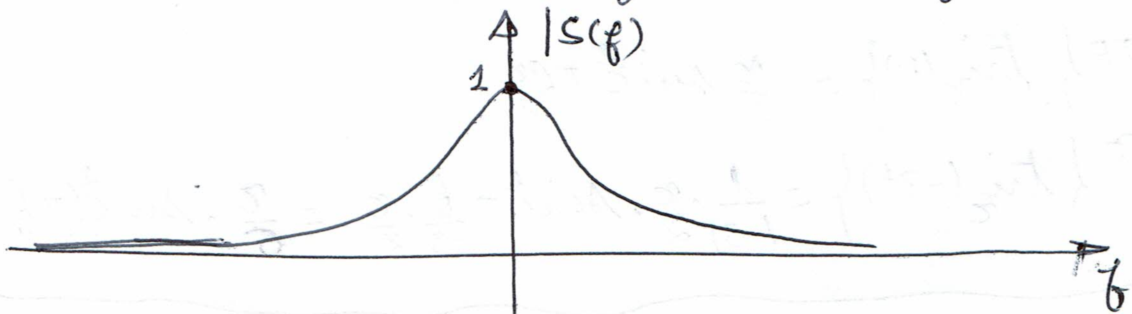
$$S(f) = \int_0^{+\infty} 2 . e^{-(2+j2\pi f)t} dt$$

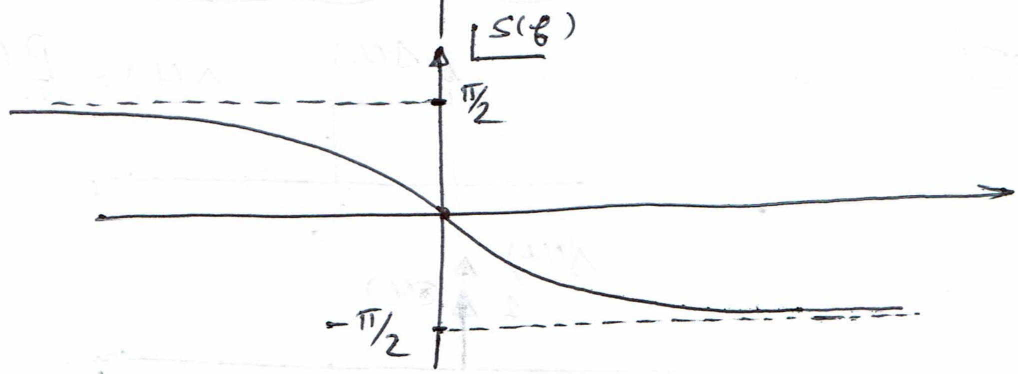
$$S(f) = -\frac{2\times 1}{2+j2\pi f}\, e^{-2t}\cdot e^{-j2\pi f t}\Big|_0^{+\infty}$$

$$\boxed{S(f) = \frac{2}{2+j2\pi f}}$$, don $TF\left\{\underset{\alpha>0}{e^{-\alpha t} u(t)}\right\} = \frac{1}{\alpha + j2\pi f}$.

Spectre d'amplitude $|S(f)| = \dfrac{2}{\sqrt{4+4\pi^2 f^2}}$

Spectre de phase $\angle S(f) = 0 - \text{Arctg}\,\pi f = -\text{Arctg}\,\pi f$.





Exo4 : par application des proprietés de la TF on a :

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* $TF\{5\,P_\tau(t)\} = 5.\tau\,\text{sinc}\,\pi f\tau$.
* $TF\{P_\tau(t-2)\} = \tau.\,\text{sinc}\,\pi f\tau \cdot e^{-j2\pi\times 2\times f} = \tau\,\text{sinc}\,\pi f\tau\, e^{-j4\pi f}$.
* $TF\{4P_\tau(t+3)\} = 4\tau.\,\text{sinc}\,\pi f\tau.\, e^{j6\pi f}$.
* $TF\{P_\tau(2t)\} = \frac{1}{|2|}\tau\cdot\text{sinc}(\pi f/_2\cdot\tau) = \frac{\tau}{2}.\,\text{sinc}\,\pi f\frac{\tau}{2}$.
* $TF\{2\,\text{sinc}^2(\pi t/_2)\} = TF\{4\times\frac{1}{2}.\,\text{sinc}^2\pi.t.\frac{1}{2}\} = 4\,\text{tri}_1(f)$

La fonction tri(f) et paire alors.

$$TF\{2\,\text{sinc}^2(\pi t/_2)\} = 4\,\text{tri}_1(f).$$

2) * $TF\{u(t)\} = \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$

* $TF\{4u(t)\cdot e^{j5t}\} = TF\{4u(t)\cdot e^{j2\pi\cdot\frac{5}{2\pi}t}\} = \left[\frac{1}{2}\delta(f-\frac{5}{2\pi}) + \frac{1}{j2\pi(f-\frac{5}{2\pi})}\right]\times 4$

* $TF\{4u(t)\cdot e^{j5t}\} = 2\delta(f-\frac{5}{2\pi}) + \frac{2}{j\pi(f-\frac{5}{2\pi})}$

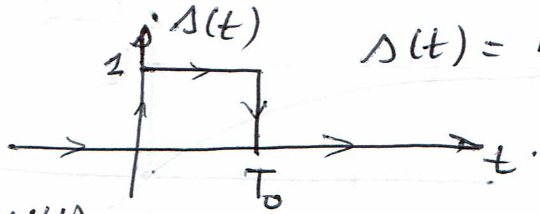
* $TF\{2u(t-2)\} = \left[\frac{1}{2}\delta(f)\cdot e^{-j4\pi f} + \frac{1}{j2\pi f}\cdot e^{-j4\pi f}\right]\times 2$

$TF\{2u(t-2)\} = \delta(f)\cdot e^{-j4\pi f} + \frac{1}{j\pi f}\cdot e^{-j4\pi f}$

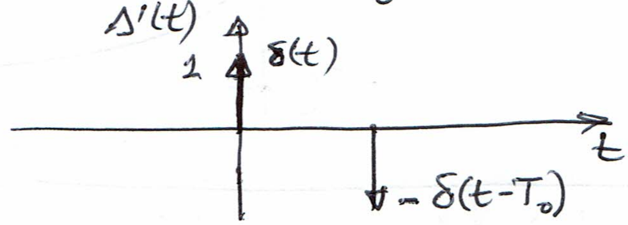
3) $TF\{tri_\tau(t)\} = \frac{\tau}{2}\,\mathrm{sinc}^2\,\pi f\frac{\tau}{2}$

$TF\{tri_\tau(-3t)\} = \frac{1}{|-3|}\cdot\frac{\tau}{2}\cdot\mathrm{sinc}^2\,\pi\frac{f}{-3}\cdot\frac{\tau}{2} = \frac{\tau}{6}\cdot\mathrm{sinc}^2(\pi f\,\tau/6)$

Exos :



$s(t) = \Pi_{T_0}(t-\frac{T_0}{2})$



$s'(t) = \delta(t) - \delta(t-T_0)$

$TF\{s'(t)\} = 1 - e^{-j2\pi f T_0}$

d'autre part on a : $TF\{s'(t)\} = j2\pi f\cdot S(f)$, Alors

$S(f) = \frac{TF\{s'(t)\}}{j2\pi f} = \frac{1-e^{-j2\pi f T_0}}{j2\pi f} = \frac{(e^{j\pi f T_0} - e^{-j\pi f T_0})}{j2\pi f}\cdot e^{-j\pi f T_0}$

$S(f) = \left(\frac{\sin \pi f T_0}{\pi f}\cdot e^{-j\pi f T_0}\right)\times\frac{T_0}{T_0}$

Alors $S(f) = T_0\cdot\mathrm{sinc}\,\pi f T_0\cdot e^{-j\pi f T_0}$

Exo6 : Calcul de l'énergie du signal

$s(t) = AT_0 \operatorname{sinc} 2\pi f_0 (t - T_0)$.

par application du théorème de Parceval

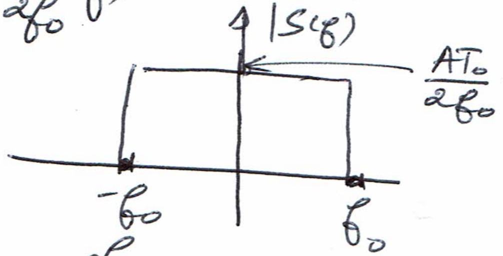
$$E = \int_{-\infty}^{+\infty} |s(t)|^2 dt = \int_{-\infty}^{+\infty} |S(f)|^2 df.$$

Alors

$$S(f) = TF\left\{ AT_0 \operatorname{sinc} 2\pi f_0 (t - T_0) \right\} = TF\left\{ \frac{AT_0}{2f_0} \cdot 2f_0 \cdot \operatorname{sinc}[\pi (t - T_0) \times 2f_0] \right\}$$

$$S(f) = \frac{AT_0}{2f_0} \cdot P_{2f_0}(f) \cdot e^{-j2\pi f T_0}.$$

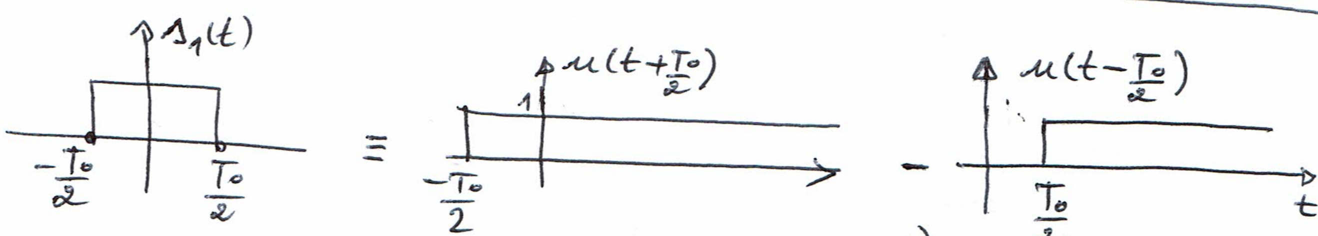
$$|S(f)| = \frac{AT_0}{2f_0} \cdot P_{2f_0}(f) \quad \text{car } \left|e^{-j2\pi f T_0}\right| = 1.$$



$$E = \int_{-\infty}^{+\infty} |S(f)|^2 df = \int_{-f_0}^{f_0} \frac{A^2 T_0^2}{4 f_0^2} df = \frac{A^2 T_0^2}{4 f_0^2} \cdot f \Big|_{-f_0}^{f_0}$$

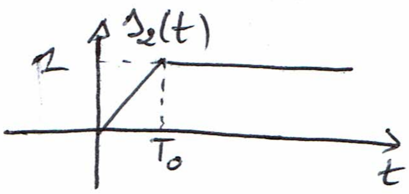
$$E = \frac{A^2 \cdot T_0^2}{2 f_0} \text{ Joules}$$

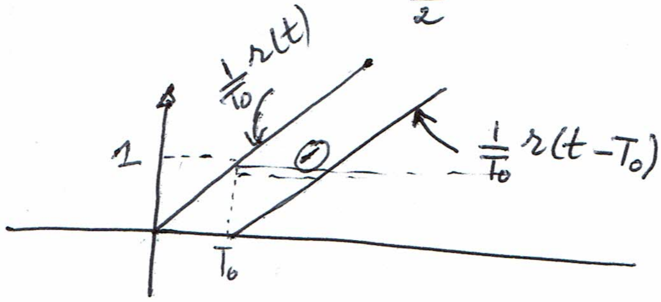
Exo7



1) $s_1(t) = u(t + \frac{T_0}{2}) - u(t - \frac{T_0}{2})$.

2)





$$s_2(t) = \frac{1}{T_0} r(t) - \frac{1}{T_0} r(t - T_0)$$

, avec $r(t) = t\, u(t)$.

Exo8 :

$s(t) = rect_T(t) * rect_T(t)$

1) par applictrication de la théoreme de la convolution

$TF\{s(t)\} = TF\{rect_T(t)\} \cdot TF\{rect_T(t)\}$

$S(f) = T sinc(\pi f T) \cdot T sinc \pi f T$, Alors .

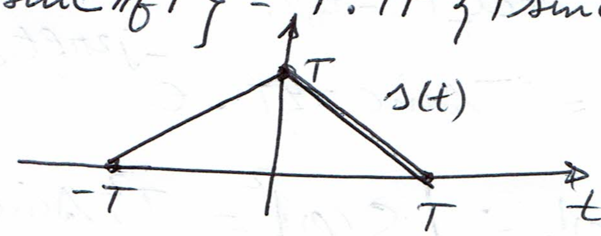
$$\boxed{S(f) = T^2 \cdot sinc^2 \pi f T}$$

2) par application de la TF^{-1} on a :

$s(t) = TF^{-1}\{S(f)\} = TF^{-1}\{T^2 sinc^2 \pi f T\}$

$s(t) = TF^{-1}\{T \cdot T sinc^2 \pi f T\} = T \cdot TF^{-1}\{T sinc^2 \pi f T\}$

$s(t) = T \cdot tri_{2T}(t)$



Exo9 :

1) $t^2 * \delta(t-\frac{1}{2}) = \int_{-\infty}^{+\infty} \tau^2 \cdot \delta(t-\tau-\frac{1}{2}) d\tau = \tau^2 \Big|_{\tau = t-\frac{1}{2}} = (t-\frac{1}{2})^2$

2) $rect_{T_0}(t) * \delta(t) = rect_{T_0}(t)$, Car $\delta(t)$: est l'element neutre pour la convolution

3) $\delta(t-t_0) * \delta(t-t_0) = \int_{-\infty}^{+\infty} \delta(\tau-t_0) \cdot \delta(t-\tau-t_0) d\tau$

$= \delta(\tau - t_0)\Big|_{\tau = t-t_0} = \delta(t-2t_0)$

Exo 10 : Calcul de la F.A.C

1) $s_1(t) = rect_T(t)$

$S_1(f) = TF\{rect_T(t)\} = T.\,sinc\,\pi f T$

$TF\{R_{s_1}(\tau)\} = |S_1(f)|^2 = T^2.\,sinc^2\pi f T = T.\,T\,sinc^2\pi f T$

Alors la fonction d'autocorrelation sera.

$R_{s_1}(\tau) = TF^{-1}\{|S_1(f)|^2\} = TF^{-1}\{T.\,T\,sinc^2\pi f T\}$.

$\boxed{R_{s_1}(\tau) = T.\,tri_{2T}(\tau)}$

2) $s_2(t) = rect_T(t-t_0)$.

$S_2(f) = T\,sinc\,\pi f T \cdot e^{-j2\pi f t_0}$.

$TF\{R_{s_2}(\tau)\} = |S_2(f)|^2 = T^2.\,sinc^2\pi f T \left(\text{Car } |e^{-j2\pi f t_0}| = 1\right)$

donc

$R_{s_2}(\tau) = TF^{-1}\{|S_2(f)|^2\} = TF^{-1}\{T^2 sinc^2\pi f T\}$.

$R_{s_2}(\tau) = T.\,tri_{2T}(\tau)$

On remarque que $F.A.C\{rect_T(t)\} = FAC\{rect_T(t-t_0)\}$

Alors on peut conclure que le décalage n'influe pas sur la F.A.C

càd $\boxed{F.A.C\{s(t)\} = F.A.C\{s(t\pm t_0)\}}$