Exercise Sheet 1

Exercise 1

Statement	**Proposition?**	**True/False or Reason**
12 is an even number.	Yes	True
$x + 3 = 7$	No	Contains a variable with unspecified value
Close the window.	No	Imperative command
The program has finished execution.	Yes	True/False depends on context
5 is a prime number.	Yes	True
Is the network connected?	No	Question
$10 < 3$	Yes	False
Stop the program!	No	Imperative command
Oran is in Egypt.	Yes	False
$y > 0$	No	Contains a variable with unspecified value

Exercise 02

Check whether the sentences can be translated into propositional logic. If yes, indicate the propositional variables and translation.

1. Imad enrolled in second year computer science.
   Propositional variable: P
   Translation: P

2. ChatGPT is an AI text generator developed by OpenAI.
   Propositional variable: Q
   Translation: Q

3. If I turn off the laptop then the stored data are lost.
   Propositional variables: R: "I turn off the laptop", S: "The stored data are lost"
   Translation: $R \Rightarrow S$

4. Salim comes to the party if and only if Amine comes and Kamel does not come.
   Propositional variables: A: "Salim comes", B: "Amine comes", C: "Kamel comes"
   Translation: $A \Leftrightarrow (B \wedge \neg C)$

5. It is not true that Salim will come if Amine or Kamel comes.
   Propositional variables: A: "Salim comes", B: "Amine comes", C: "Kamel comes"
   Translation: $\neg((B \vee C) \Rightarrow A)$

6. Salim comes to the party on condition that Kamel does not come, but if Kamel comes, Amine does not come.
   Propositional variables: A: "Salim comes", B: "Amine comes", C: "Kamel comes"
   Translation: $(\neg C \Rightarrow A) \wedge (C \Rightarrow \neg B)$

7. The server is not only running, but it is also connected to the network.
   Propositional variables: R: "Server is running", N: "Server is connected to network"
   Translation: $R \wedge N$

8. If Salim is brave and intelligent then John is not happy.
   Propositional variables: S: "Salim is brave", T: "Salim is intelligent", J: "John is happy"
   Translation: $(S \wedge T) \Rightarrow \neg J$

9. Ask a question.
   Not a proposition (imperative, cannot be true or false)

10. In case of bad weather, or if too many participants are absent, the session will be postponed.
    Propositional variables: B: "Bad weather", A: "Too many participants are absent", P: "Session will be postponed"
    Translation: $(B \vee A) \Rightarrow P$

11. Computers facilitate administration but also teaching.
    Propositional variables: C: "Computers facilitate administration", T: "Computers facilitate teaching"
    Translation: $C \wedge T$

12. What is your email address?
    Not a proposition (question, cannot be true or false)

13. Computers make administration easier and may or may not make teaching easier.
    Propositional variables: A: "Computers make administration easier", T: "Computers make teaching easier"
    Translation: $A \wedge (T \vee \neg T)$

14. You did a really good job!
    Not a proposition (exclamation, cannot be true or false)

15. Five plus two.
    Not a proposition (arithmetic expression without assertion)

16. An empty battery is a sufficient condition for the laptop to turn off.
    Propositional variables: E: "Battery is empty", L: "Laptop turns off"
    Translation: $E \Rightarrow L$

17. Installing the software is not a sufficient condition for it to run correctly.
    Propositional variables: I: "Software is installed", R: "Software runs correctly"
    Translation: $\neg(I \Rightarrow R)$

Exercise 03

Put parentheses according to connector precedence.

1. $p \wedge q \Rightarrow r \vee \neg s$
   With parentheses: $(p \wedge q) \Rightarrow (r \vee (\neg s))$

2. $\neg p \vee \neg q \wedge r$
   With parentheses: $(\neg p) \vee ((\neg q) \wedge r)$

3. $\neg p \Leftrightarrow q$
   With parentheses: $((\neg p) \Leftrightarrow q)$

4. $\neg p \Rightarrow \neg q \wedge r$
   With parentheses: $(\neg p) \Rightarrow ((\neg q) \wedge r)$

5. $\neg p \Rightarrow \neg q \Leftrightarrow r$
   With parentheses: $((\neg p) \Rightarrow (\neg q)) \Leftrightarrow r$

Exercise 04

By interpreting:

- p: "I leave"
- q: "you stay"
- r: "there's no one"

1. $p \wedge \neg q \Rightarrow r$
   Natural language:

   > If I leave and you do not stay, then there is no one.

2. $\neg p \vee \neg q \Rightarrow \neg r$
   Natural language:

   > If I do not leave or you do not stay, then there is someone.

Exercise 05

A formula is a WFF (well-formed formula) if it is built only from propositional variables and logical connectives with correct parentheses.

1. $((p \Rightarrow q) \wedge \neg p) \Rightarrow \neg q$
   WFF — correctly built using $\Rightarrow, \wedge, \neg$ and parentheses.

2. $\neg(p \Rightarrow (q = p))$
   Not WFF — the symbol $=$ is not a propositional connective.

3. pq
   Not WFF — two variables written together with no connective.

4. $\neg(\leq (q \wedge p))$
   Not WFF — the symbol $\leq$ is not a propositional connective.

5. $p - q$
   Not WFF — the symbol $-$ is arithmetic, not logical.

6. $(p \Rightarrow \neg q) \Leftrightarrow r$
   WFF — valid use of $\Rightarrow$, $\neg$, and $\Leftrightarrow$.

Exercise 06

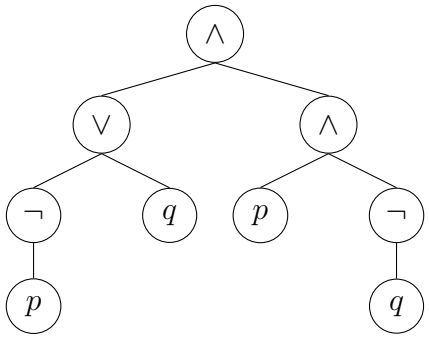
Depth = number of branches from root to farthest leaf
Complexity = number of connectives

1) $(\neg p \vee q) \wedge (p \wedge \neg q)$

Subformulas:

$$p,\ q,\ \neg p,\ \neg q,\ (\neg p \vee q),\ (p \wedge \neg q),\ (\neg p \vee q) \wedge (p \wedge \neg q)$$

Depth = 3
Complexity = 5

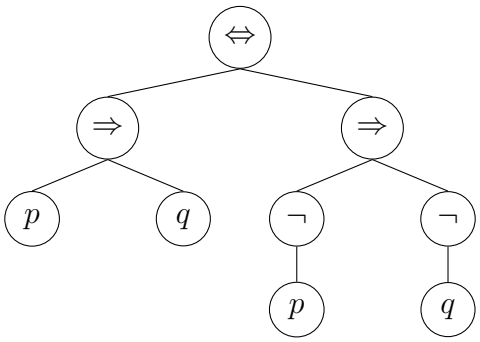


2) $(p \Rightarrow q) \Leftrightarrow (\neg p \Rightarrow \neg q)$

Subformulas:

$$p, q, \neg p, \neg q, (p \Rightarrow q), (\neg p \Rightarrow \neg q), \text{whole formula}$$

Depth = 3
Complexity = 5

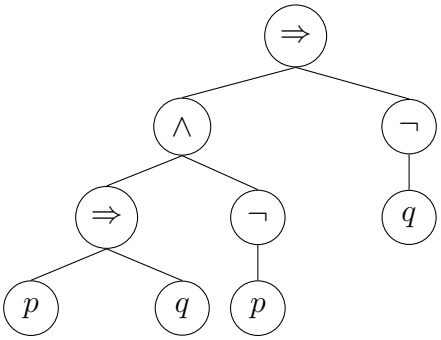


3) $[(p \Rightarrow q) \wedge \neg p] \Rightarrow \neg q$

Subformulas:

$$p, q, \neg p, \neg q, (p \Rightarrow q), ((p \Rightarrow q) \wedge \neg p), \text{whole formula}$$

Depth = 3
Complexity = 5

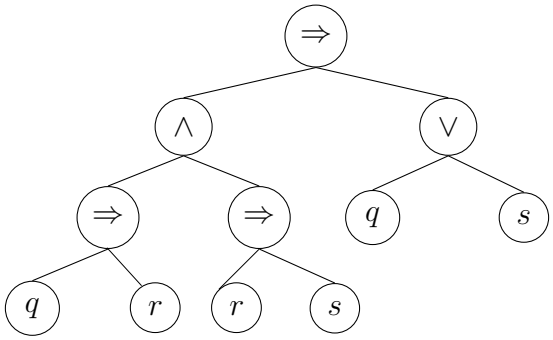


4) $((q \Rightarrow r) \wedge (r \Rightarrow s)) \Rightarrow (q \vee s)$

Subformulas:

$$q, r, s, (q \Rightarrow r), (r \Rightarrow s), ((q \Rightarrow r) \wedge (r \Rightarrow s)), (q \vee s), \text{whole formula}$$

Depth = 3
Complexity = 5



5) $(\neg p \Rightarrow q \wedge r) \Leftrightarrow ((s \Rightarrow \neg p \wedge r) \Leftrightarrow \neg q \wedge s)$

(Using precedence: $\neg$ then $\wedge$ then $\Rightarrow$)

Rewritten with parentheses:

$$(\neg p \Rightarrow (q \wedge r)) \Leftrightarrow ((s \Rightarrow (\neg p \wedge r)) \Leftrightarrow (\neg q \wedge s))$$

Subformulas:

$$p, q, r, s, \neg p,\ \neg q, (q \wedge r),\ (\neg p \wedge r),\ (\neg q \wedge s), (\neg p \Rightarrow (q \wedge r)),$$
$$(s \Rightarrow (\neg p \wedge r)), ((s \Rightarrow (\neg p \wedge r)) \Leftrightarrow (\neg q \wedge s)), \text{whole formula}$$

Depth = 5
Complexity = 10