

Exercise Sheet 4

Exercise No. 01

Write the following expressions in predicate logic:

1. Imad is Zina's father.
2. Fares is the father of Fatma and Zouhir.
3. The horse is faster than the dog.
4. They are all students.
5. Every student is registered.
6. Some students are not sick.
7. Not all men travel with John.
8. If all men travel with John, then he is happy.
9. People who have a fever and cough have the flu.
10. People who have the flu should take Tamiflu.

Exercise No. 02

Predicate language L is defined as: constants A, B ; functions f, g ; predicates P, Q, R ; variables x, y . Which of the following expressions are syntactically correct terms or formulas?

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| 1. $R(A, g(A, A))$ | 5. $\neg R(P(A), x)$ |
| 2. $\neg y \wedge P(y)$ | 6. $x \forall y (R(x, y) \Rightarrow R(y, x))$ |
| 3. $g(A, g(A, A))$ | 7. $\exists A R(A, A)$ |
| 4. $\forall x P(x)$ | 8. $\forall x Q(x, f(x), B) \Rightarrow \exists x R(A, x)$ |

Exercise No. 03

Indicate the free variables in the following formulas:

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|--|---|
| 1. $P(x) \wedge R(y, a)$ | 5. $\forall x P(x) \Rightarrow Q(y)$ |
| 2. $\exists y (Q(y) \wedge P(x))$ | 6. $P(x) \Rightarrow \exists x Q(x)$ |
| 3. $\forall x \exists y R(x, f(y))$ | 7. $\exists x (P(x) \Rightarrow Q(x))$ |
| 4. $\forall x (P(x) \Rightarrow Q(x))$ | 8. $\forall x (P(x) \Rightarrow \exists y (Q(y) \wedge R(x, y)))$ |

Exercise No. 04

Construct the construction tree and perform the negation of the following formulas:

1. $\forall x \exists y (P(x) \vee Q(y))$
2. $\forall x (P(x) \Rightarrow \exists y (Q(y) \wedge R(x, y)))$
3. $\forall x \exists y (R(x, y) \Rightarrow (R(y, x) \vee Q(f(A), g(A, x))))$

Exercise No. 05

Consider the following formulas:

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|----------------------------------|---|
| 1. $\exists x P(x, Zero)$ | 5. $\exists y \forall x P(x, y)$ |
| 2. $\forall x P(x, x)$ | 6. $\forall y \exists x P(x, y)$ |
| 3. $\forall x \exists y P(x, y)$ | 7. $\exists x \forall y \neg E(y, x) \Rightarrow P(x, y)$ |
| 4. $\exists x \forall y P(x, y)$ | 8. $\forall x (P(x, y) \wedge \neg P(x, y))$ |

Determine whether the above formulas are true or false for the following interpretation:
 $D = \mathbb{N}$, $P(x, y) \equiv x > y$, $E(x, y) \equiv x = y$, $Zero = 0$.

Exercise No. 06

Convert the following formulas into Prenex and Skolem forms:

- $(\forall x P(x)) \Rightarrow Q(y)$
- $(\exists x (\neg F(x) \vee G(x))) \wedge (\exists x F(x) \wedge \forall x \neg G(x))$
- $\forall x P(x) \wedge \exists y Q(y) \Rightarrow \exists y (P(y) \wedge Q(y))$
- $\forall x [(\forall y P(x) \vee \forall z Q(z, y)) \Rightarrow \neg \forall y R(x, y)]$