Exercise Sheet 2

Exercise No. 01

Consider the formulas: $F_1 = p \wedge (\neg q \Rightarrow (q \Rightarrow p))$, $\quad F_2 = (p \vee q) \Leftrightarrow (\neg p \Rightarrow \neg q)$.
and let I be an interpretation.

1. Compute the truth tables of F_1 and F_2.
2. Determine the set of models of the formula F_2.
3. Determine, if possible, the truth values of F_1 and F_2 in each of the following cases:
   (a) $I(p) = 0$ and $I(q) = 1$
   (b) $I(p) = 0$
   (c) $I(q) = 1$
   (d) No information about $I(p)$ and $I(q)$

Exercise No. 02

Check that the following formulas are tautologies:

$F_1 = \neg(p \Rightarrow q) \Rightarrow \neg q$ $\qquad$ $F_2 = (p \Rightarrow q) \vee (q \Rightarrow p)$

1. Using equivalent transformations.
2. Using truth table.

Exercise No. 03

Check the following logical equivalences using truth tables:

1. $F_1 = p \vee (p \wedge q)$, and $\quad F_2 = p$
2. $F_3 = p \wedge (q \vee r)$, and $\quad F_4 = (p \wedge q) \vee (p \wedge r)$
3. $F_5 = (p \Rightarrow q) \Rightarrow r$, and $\quad F_6 = p \Rightarrow (q \Rightarrow r)$

Exercise No. 04

I. Convert to CNF & DNF using truth table:

1. $F_1 = (p \wedge q) \Leftrightarrow (\neg p \Rightarrow q)$
2. $F_2 = (\neg p \Rightarrow q) \Rightarrow (q \Rightarrow \neg r)$
3. $F_3 = (\neg p \vee q) \vee (p \Rightarrow r)$

II. Convert to CNF using equivalences:

1. $F_4 = \neg(\neg p \vee q) \wedge (\neg r \Rightarrow \neg s)$
2. $F_5 = (p \wedge q) \vee (\neg p \vee q)$
3. $F_6 = (p \wedge \neg q) \Rightarrow \neg r$

Exercise No. 05

Verify validity, satisfiability, or unsatisfiability using truth tables:

1. $F_1 = (p \Leftrightarrow q) \Rightarrow ((p \Rightarrow q) \vee (q \Rightarrow p))$
2. $F_2 = (p \Rightarrow q) \Rightarrow (q \vee \neg p)$
3. $F_3 = p \wedge \neg(q \vee p)$
4. $F_4 = (p \wedge q) \wedge \neg p$
5. $F_5 = (p \Rightarrow \neg q) \wedge q$