MOHAMED BOUDIAF UNIVERSITY - M'SILA	Academic Year 2025-2026
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE	**L1:** Computer Science
DEPARTMENT OF COMPUTER SCIENCE	Duration: 2 H 00 min

Correction Final Exam of Mathematical Logic (Normal Session)

Exercise No. 01 (6×0.5 pt)

No.	Statement	Answer	Justification (if False)
1	We can translate "$x + 10 > 5$" to an assertion in propositional logic.	False	The expression contains the variable x.
2	The formula $p \wedge (\neg q \Rightarrow \neg(\neg q \Rightarrow p))$ is a well-formed formula.	True	–
3	For a given formula, several trees can be found.	False	For every well-formed formula, the parsing tree is unique.
4	The CNF for a given formula is not unique.	True	–
5	The formula $(p \vee r \wedge s)$ is a positive clause.	False	A positive clause must be a disjunction of literals only.
6	The existential quantifier $\exists$ is one of the components of a Skolem form.	False	In Skolem form, existential quantifiers are eliminated using Skolem constants or functions.

Exercise No. 02 (4 marks)

1) Truth Tables

$\mathbf{F}_1 : p \wedge (\neg q \Rightarrow (q \Rightarrow p))$

p	q	$q \Rightarrow p$	$\neg q$	$\neg q \Rightarrow (q \Rightarrow p)$	F_1	Maxterms
0	0	1	1	1	0	$p \vee q$
0	1	0	0	1	0	$p \vee \neg q$
1	0	1	1	1	1	
1	1	1	0	1	1	

......(1 pt)

$\mathbf{F}_2 : (p \vee q) \Leftrightarrow (\neg p \Rightarrow \neg q)$

p	q	$p \vee q$	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$	F_2
0	0	0	1	1	1	0
0	1	1	1	0	0	0
1	0	1	0	1	1	1
1	1	1	0	0	1	1

......(1 pt)

$\text{Mod}(F_2) = \{(I(p) = 1, I(q) = 0), (I(p) = 1, I(q) = 1)\}$......(0.5 pt)

3) Truth values

- $I(p) = 0, I(q) = 1$: $F_1 = 0,\ F_2 = 0$ ......(0.25 pt)
- $I(p) = 0$: $F_1 = 0,\ F_2 = 0$ ......(0.25 pt)
- $I(q) = 1$: depends on p ......(0.25 pt)
- No information: depends on p ......(0.25 pt)

4) CNF of F_1

$$\boxed{(p \vee q) \wedge (p \vee \neg q) \equiv p}\text{......(0.5 pt)}$$

Exercise No. 03 (5 marks)

1. Formalization in propositional logic

(a) "If I do not pass, then I did not study."

Let:

$$p: \text{ I pass,} \qquad q: \text{ I study}$$

Then:

$$\boxed{\neg p \Rightarrow \neg q}\text{......(0.5 pt)}$$

(b) "Amine is Algerian or if Amine is not Algerian then Salim is Tunisian."

Let:

$$p: \text{ Amine is Algerian}$$

$$q: \text{ Salim is Tunisian}$$

Then:

$$\boxed{p \vee (\neg p \Rightarrow q)}\text{......(0.5 pt)}$$

(c) "Amine comes to the exam iff Brahim comes and Salim does not come."

Let:

$$a: \text{ Amine comes}$$

$$b: \text{ Brahim comes}$$

$$s: \text{ Salim comes}$$

Then:

$$\boxed{a \Leftrightarrow (b \wedge \neg s)}\text{......(0.5 pt)}$$

2. Formalization in predicate logic

(a) "There exists a smart student."

$$\boxed{\exists x(Student(x) \land Smart(x))} \text{......(0.5 pt)}$$

(b) "Every student takes at least one course."

$$\boxed{\forall x\big(Student(x) \Rightarrow \exists y(Course(y) \land Takes(x,y))\big)} \text{......(0.5 pt)}$$

(c) "There is a printer which is not used by any student."

$$\boxed{\exists x\Big(Printer(x) \land \forall y(Student(y) \Rightarrow \neg Uses(y,x))\Big)} \text{......(0.5 pt)}$$

3. Negations

Negation of 1.(c)

$$\neg[a \Leftrightarrow (b \land \neg s)]$$

Equivalent form:

$$\boxed{(a \land (\neg b \lor s)) \lor (b \land \neg s \land \neg a)} \text{ ......(0.75 pt)}$$

Negation of 2.(c)

$$\neg\exists x\Big(Printer(x) \land \forall y(Student(y) \Rightarrow \neg Uses(y,x))\Big)$$

$$\equiv \forall x\Big(\neg Printer(x) \lor \exists y(Student(y) \land Uses(y,x))\Big) \text{ ......(0.75 pt)}$$

4. Closed formula

The formula:

$$\forall x\big(Student(x) \Rightarrow \exists y(Course(y) \land Takes(x,y))\big)$$

is a closed formula because all variables are bounded (quantified). ......(0.5 pt)

Exercise No. 04 (8 marks)

Given:

$$F_1 : p \Rightarrow q \land r, \qquad F_2 : (p \Rightarrow q) \Rightarrow r, \qquad F_3 : p \land q \Rightarrow r, \qquad F_4 : (p \land q) \Rightarrow (q \Rightarrow r)\,.$$

1. Valid, satisfiable or unsatisfiable

p	q	r	F_1	F_2	F_3	$F_1 \Rightarrow F_2$
0	0	0	1	0	1	0
0	0	1	1	1	1	1
0	1	0	1	0	1	0
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	0	0	0	1
1	1	1	1	1	1	1

......(1.5 pt)

- F_1 is satisfiable but not valid, and it isn't unsatisfiable. ......(0.25 pt)
- F_2 is satisfiable but not valid, and it isn't unsatisfiable. ......(0.25 pt)
- F_3 is satisfiable but not valid, and it isn't unsatisfiable. ......(0.25 pt)
- $F_1 \Rightarrow F_2$ is satisfiable but not valid, and it isn't unsatisfiable. ......(0.25 pt)

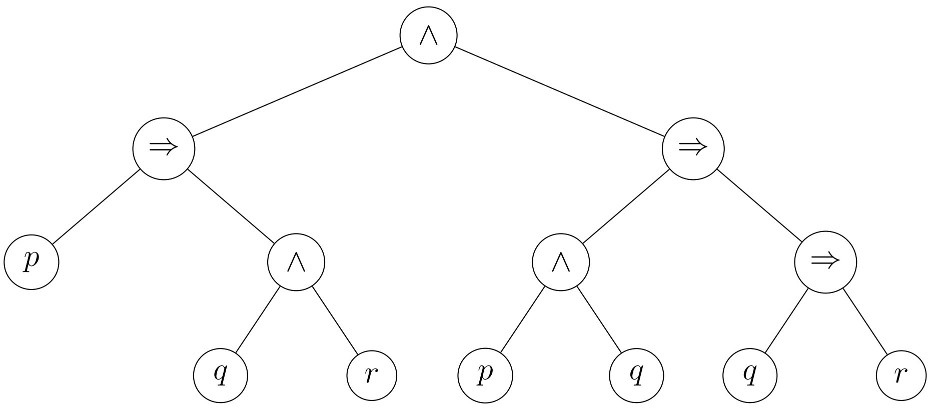
2. Parsing tree and sub-formulas

Formula:

$$F_1 \wedge F_4$$

Sub-formulas:

$$\{p,\ q,\ r,\ q \wedge r,\ p \Rightarrow (q \wedge r), p \wedge q,\ q \Rightarrow r, (p \wedge q) \Rightarrow (q \Rightarrow r), F_1 \wedge F_4\}$$ ......(0.75 pt)



......(0.75 pt)

Depth:

$\boxed{3}$ ......(0.25 pt)

Complexity:

$\boxed{6}$ ......(0.25 pt)

3. Check whether $F_1 \models F_2$

Since:

$$F_1 \Rightarrow F_2$$

is not a tautology, we conclude:

$\boxed{F_1 \not\models F_2}$ ......(0.5 pt)

4. Prove that $F_3 \Rightarrow F_4$ is a tautology

$$F_3 = (p \land q) \Rightarrow r$$

$$F_4 = (p \land q) \Rightarrow (q \Rightarrow r)$$

Now:

$$q \Rightarrow r \equiv \neg q \lor r$$

Thus:

$$F_4 \equiv \neg(p \land q) \lor (\neg q \lor r)$$

$$\equiv \neg p \lor \neg q \lor r$$

But:

$$F_3 \equiv \neg(p \land q) \lor r \equiv \neg p \lor \neg q \lor r$$

Hence:

$$F_3 \equiv F_4$$

Therefore:

$$F_3 \Rightarrow F_4 \equiv T$$

So it is a tautology. ......(1 pt)

5. Convert into CNF

Formula:

$$(\neg p \Rightarrow q) \Rightarrow (q \Rightarrow \neg r)$$

Eliminate implications:

$$(p \lor q) \Rightarrow (\neg q \lor \neg r)$$

$$\equiv \neg(p \lor q) \lor (\neg q \lor \neg r)$$

$$\equiv (\neg p \land \neg q) \lor (\neg q \lor \neg r)$$

CNF:

$$\boxed{(\neg p \lor \neg q \lor \neg r) \land (\neg q \lor \neg r)} \text{ ......(1 pt)}$$

6. Resolution proof

Given:

$$\Sigma = \{r,\ \neg q \Rightarrow \neg r,\ p \lor \neg q \lor \neg s, \neg p \lor \neg q \lor \neg s,\ s \lor \neg r\}$$

First convert:

$$\neg q \Rightarrow \neg r \equiv q \lor \neg r$$

Clauses:

$$C_1 = r$$

$$C_2 = q \lor \neg r$$

$$C_3 = p \lor \neg q \lor \neg s$$

$$C_4 = \neg p \lor \neg q \lor \neg s$$

$$C_5 = s \vee \neg r$$

Resolution steps:
From C_1 and C_2:

$$q$$

From C_1 and C_5:

$$s$$

From q and C_3:

$$p \vee \neg s$$

Using s:

$$p$$

From q and C_4:

$$\neg p \vee \neg s$$

Using s:

$$\neg p$$

Finally:

$$p, \ \neg p \Rightarrow \square$$

Therefore:

$$\boxed{\Sigma \text{ is unsatisfiable}} \text{ ......(1 pt)}$$