Mathematical Logic

Lecture 3: Semantics - Part 1

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Level: First Year LMD — Licence Computer Science

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Interpretation in Propositional Logic

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- **Example:** Let $F : p \Rightarrow (q \wedge r)$.

  $\{I(p) = 1, I(q) = 0, I(r) = 1\}$: be an interpretation of the formula F

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- A formula with n atomic variables admits 2^n interpretations.
- **Example:** The formula $F : p \Rightarrow (q \wedge r)$ has three atomic variables $\{p, q, r\}$, so it admits $2^3 = 8$ interpretations.
- **Model:** An interpretation I is a model of F if $I(F) = 1$.

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- The truth value of the formula depends on the truth values of the subformulas, and implicitly the atomic variables.
- If the formula consists of n atomic variables, we obtain 2^n combinations, corresponding to 2^n lines of the table.
- Truth tables allow us to define and visualize the logical connectives in propositional logic.

Truth Tables

Truth table represents all interpretations of a formula based on logical connectives.

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$
0	0	0	0	1	1	1
0	1	0	1	1	1	0
1	0	0	1	0	0	0
1	1	1	1	0	1	1

Remarks on Logical Connectives

- **Conjunction** $p \wedge q$ is **true** if and only if p and q are both true.
  - Example: $p = 1, q = 1 \Rightarrow p \wedge q = 1$
  - Example: $p = 1, q = 0 \Rightarrow p \wedge q = 0$

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- **Disjunction** $p \vee q$ is **true** if and only if either p or q is true.
  - Example: $p = 0, q = 1 \Rightarrow p \vee q = 1$
- **Implication** $p \Rightarrow q$ is false if and only if p is true and q is false.
  - Example: $p = 1, q = 0 \Rightarrow p \Rightarrow q = 0$
  - Otherwise it is **true**.

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  - Example: $p = 0, q = 1 \Rightarrow p \vee q = 1$
- **Implication** $p \Rightarrow q$ is false if and only if p is true and q is false.
  - Example: $p = 1, q = 0 \Rightarrow p \Rightarrow q = 0$
  - Otherwise it is **true**.
- **Biconditional** $p \Leftrightarrow q$ is false if and only if p and q are different.
  - Example: $p = 0, q = 1 \Rightarrow p \Leftrightarrow q = 0$
  - Example: $p = 1, q = 1 \Rightarrow p \Leftrightarrow q = 1$

Example with Subformulas

- Let $F : (p \Rightarrow (q \wedge r))$
- Subformulas: $q \wedge r$, $p \Rightarrow (q \wedge r)$

p	q	r	$q \wedge r$	$F : (p \Rightarrow (q \wedge r))$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

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Construct the truth table of the formula:

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0	1	0	1	0	1
1	0	1	1	1	1
1	1	0	1	0	0

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Set of models of F (interpretations making F true):

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0	0	1	0	0	1
0	1	0	1	0	1
1	0	1	1	1	1
1	1	0	1	0	0

Set of models of F (interpretations making F true):

$\{I(p) = 0 \text{ and } I(q) = 0,\ I(p) = 0 \text{ and } I(q) = 1,\ I(p) = 1 \text{ and } I(q) = 0\}$

Validity and Satisfiability

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- **Tautology (Valid)**: true in all interpretations.
- **Antilogy / Unsatisfiable**: false in all interpretations.
- **Satisfiable**: true in at least one interpretation.
- **Invalid**: false in at least one interpretation.

Definition: A propositional formula is said to be a tautology (valid) if and only if its truth value is always true, regardless of the truth values of its atomic variables.

$$F \text{ is a tautology if } \forall I, I(F) = 1$$

Example: Consider the formula

$$F : ((\neg q) \wedge (p \vee q)) \Rightarrow p$$

p	q	$(\neg q) \wedge (p \vee q)$	F
0	0	0	0
0	1	0	1
1	0	1	1
1	1	0	1

Validity and Satisfiability: Tautology

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p	q	$(\neg q) \wedge (p \vee q)$	F
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1	0	1	1
1	1	0	1

Observation: F is not a tautology because it is false for $I(p) = 0, I(q) = 0$.

Validity and Satisfiability: Tautology

Another Example:

$$P_1 : ((p \Rightarrow q) \wedge p) \Rightarrow q$$

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge p$	P_1
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

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Observation: P_1 is a tautology (valid formula) because all truth values of P_1 are 1.

Definition: A propositional formula is said to be antilogy (contradictory / unsatisfiable) if and only if its truth value is always false, regardless of the truth values of its atomic variables.

$$F \text{ is antilogy if } \forall I, I(F) = 0$$

Validity and Satisfiability: Unsatisfiable (Antilogy)

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$$F \text{ is antilogy if } \forall I, I(F) = 0$$

Example: Consider the formula

$$F : (\neg p) \wedge p$$

p	$\neg p$	$(\neg p) \wedge p$
0	1	0
1	0	0

Validity and Satisfiability: Unsatisfiable (Antilogy)

Definition: A propositional formula is said to be antilogy (contradictory / unsatisfiable) if and only if its truth value is always false, regardless of the truth values of its atomic variables.

$$F \text{ is antilogy if } \forall I, I(F) = 0$$

Example: Consider the formula

$$F : (\neg p) \wedge p$$

p	$\neg p$	$(\neg p) \wedge p$
0	1	0
1	0	0

Observation: The formula F is always false for all interpretations of p. Therefore, F is unsatisfiable / antilogy.

Validity and Satisfiability: Satisfiable Formula

Definition: A propositional formula F is said to be satisfiable if and only if there exists at least one interpretation where $I(F) = 1$.

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Example: Consider the formula

$$F : p \land q$$

p	q	$p \land q$
0	0	0
0	1	0
1	0	0
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$$F : p \wedge q$$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Observation: In the interpretation $I(p) = 1, I(q) = 1$, the formula F is true. Therefore, F is satisfiable.

Validity and Satisfiability: Invalid Formula

Definition: A propositional formula F is said to be invalid if and only if there exists at least one interpretation where $I(F) = 0$.

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$$F : \neg p \vee \neg q$$

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Observation: In the interpretation $I(p) = 1, I(q) = 1$, the formula F is false. Therefore, F is invalid.

Definition:

- Two formulas F_1 and F_2 are logically equivalent ($F_1 \equiv F_2$) if they admit the same models.
- Techniques to verify equivalence:
  1. Construct a truth table.
  2. Use equivalence chains (rules of equivalence).

Logical Equivalence

Example:

$$F_1 : p \Rightarrow q \quad \text{and} \quad F_2 : \neg p \vee q$$

p	q	$F_1 : p \Rightarrow q$	$F_2 : \neg p \vee q$
0	0	1	1
0	1	1	1
1	0	0	0
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Logical Equivalence

Example:

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0	1	1	1
1	0	0	0
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Observation: F_1 and F_2 have the same truth values in all interpretations.

$$F_1 \equiv F_2$$

Equivalent Transformations in Propositional Logic

Neutral Element:

$$\begin{cases} p \wedge T \equiv p & \text{(AND with True)} \\ p \vee F \equiv p & \text{(OR with False)} \end{cases}$$

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$$p \wedge p \equiv p, \quad p \vee p \equiv p$$

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$$\neg(\neg p) \equiv p$$

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Absorption Laws:

$$p \wedge (p \vee q) \equiv p, \quad p \vee (p \wedge q) \equiv p$$

Neutral Element:

$$\begin{cases} p \land T \equiv p & \text{(AND with True)} \\ p \lor F \equiv p & \text{(OR with False)} \end{cases}$$

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Double Negation:

$$\neg(\neg p) \equiv p$$

Absorption Laws:

$$p \land (p \lor q) \equiv p, \quad p \lor (p \land q) \equiv p$$

Commutative Laws:

$$p \land q \equiv q \land p, \quad p \lor q \equiv q \lor p$$

Equivalent Transformations in Propositional Logic

Associative Laws:

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r), \quad (p \vee q) \vee r \equiv p \vee (q \vee r)$$

Equivalent Transformations in Propositional Logic

Associative Laws:

$$(p \land q) \land r \equiv p \land (q \land r), \quad (p \lor q) \lor r \equiv p \lor (q \lor r)$$

Distributive Laws:

$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$

$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$

Associative Laws:

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$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

De Morgan's Laws:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q, \quad \neg(p \vee q) \equiv \neg p \wedge \neg q$$

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Material Implication:

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Material Implication:

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Material Equivalence:

$$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$$

Example: Verify Logical Equivalence

Determine whether the following equivalence is correct:

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Step 1: Apply Material Implication

$$\neg(p \Rightarrow \neg q) \vee r \equiv \neg(\neg p \vee \neg q) \vee r \quad \text{(Material Implication)}$$

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$$\neg(p \Rightarrow \neg q) \vee r \equiv \neg(\neg p \vee \neg q) \vee r \quad \text{(Material Implication)}$$

Step 2: Apply De Morgan's Law

$$\neg(\neg p \vee \neg q) \vee r \equiv (\neg\neg p \wedge \neg\neg q) \vee r \quad \text{(De Morgan's Law)}$$

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Step 2: Apply De Morgan's Law

$$\neg(\neg p \vee \neg q) \vee r \equiv (\neg\neg p \wedge \neg\neg q) \vee r \quad \text{(De Morgan's Law)}$$

Step 3: Apply Double Negation

$$(\neg\neg p \wedge \neg\neg q) \vee r \equiv (p \wedge q) \vee r \quad \text{(Double Negation)}$$

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$$(\neg\neg p \wedge \neg\neg q) \vee r \equiv (p \wedge q) \vee r \quad \text{(Double Negation)}$$

Conclusion: the equivalence is correct.

Exercises: Tautologies and Logical Equivalence

Exercise 1:
Prove that the following formulas are tautologies:

$$F_1 : (\neg q \wedge (p \Rightarrow q)) \Rightarrow (\neg q \vee p)$$

$$F_2 : ((p \vee q) \wedge \neg q) \Rightarrow p$$

Hint: Construct truth tables for each formula and verify that they are true for all interpretations.

Exercise 2:
Check whether the following equivalence is correct, $P \equiv Q$

$$P : (\neg q \wedge (p \vee r)) \Rightarrow p$$

$$Q : \neg(\neg p \wedge (\neg q \wedge r))$$

Hint: Apply step-by-step equivalent transformations using:

- Material Implication
- De Morgan's Laws
- Double Negation