REMINDER

Superposition Theorem

The superposition theorem applies to circuits containing independent sources (at least two independent voltage and/or current sources). Consider an electrical circuit comprising n independent sources: $S_1, S_2, \ldots, S_n$.

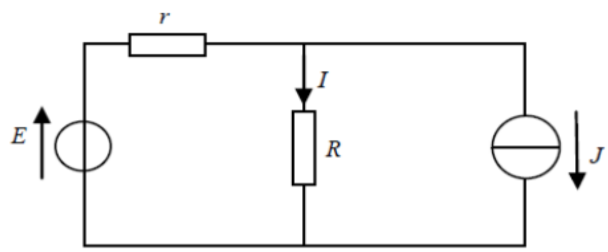
We aim to determine an electrical quantity (a current I flowing through a branch or a voltage V across an element). Let I_1 (or V_1), I_2 (or V_2), ..., I_n (or V_n) denote the values of I (or V) produced individually by each source S_j acting alone.

The other sources are deactivated (a voltage source is replaced by a short circuit, and a current source is removed, i.e., replaced by an open circuit).

The total current I (or voltage V) is equal to the algebraic sum of the currents I_j (or voltages V_j).

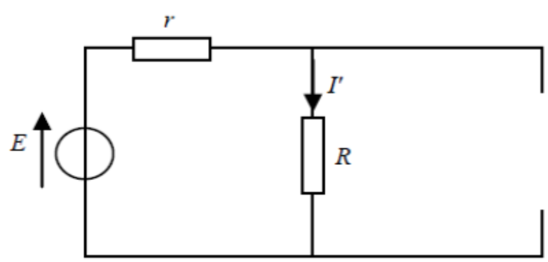
Example

Determine the expression of the current I flowing through the resistance R.



Step 1: Calculation of I when $J = 0$

Let I' be the current in the branch containing R when the independent current source J is deactivated (removed).

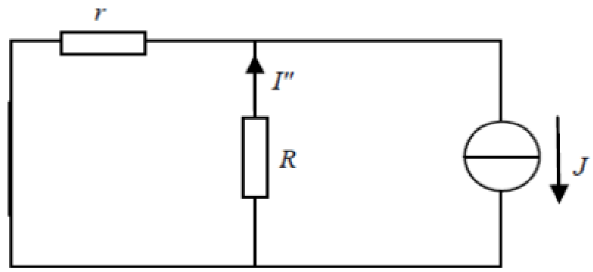


It is evident that:

$$I' = \frac{E}{r + R}$$

Step 2: Calculation of I when $E = 0$

Let I'' be the current in the branch containing R when the independent voltage source E is deactivated (short-circuited).



Applying the current divider rule, we obtain:

$$I'' = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{r}} J$$

Step 3

The total current I is the algebraic sum of the currents I' and I'', that is:

$$I = I' - I''$$

The negative sign associated with I'' indicates that it flows through R in the direction opposite to I.

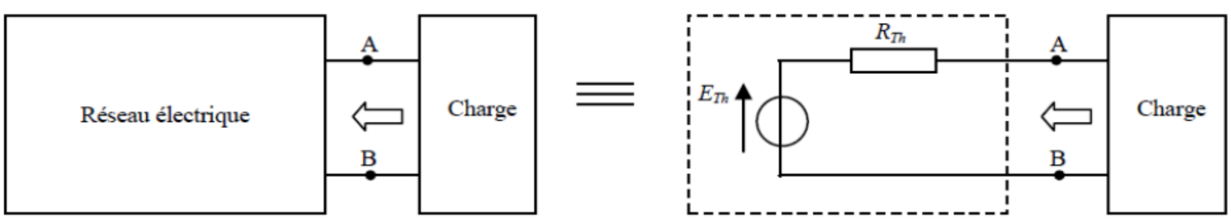
Finally, we obtain:

$$I = \frac{E}{r + R} - \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{r}} J$$

Thévenin's Theorem

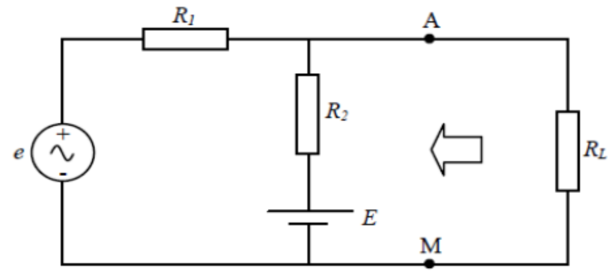
A linear circuit placed between two terminals A and B can be replaced by an equivalent Thévenin generator consisting of a voltage source E_{Th} in series with a Thévenin internal resistance (or impedance) R_{Th}.

- E_{Th} is equal to the open-circuit voltage (load disconnected) measured between terminals A and B.
- R_{Th} is the resistance seen between terminals A and B when all independent sources are deactivated (voltage sources are short-circuited and current sources are opened).



Example

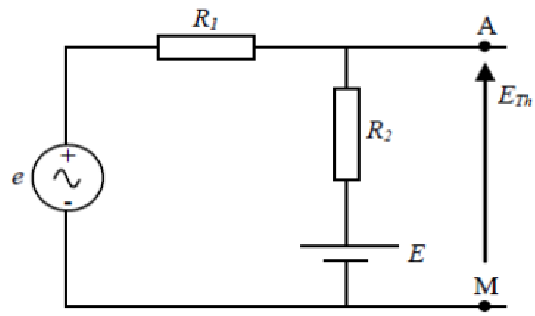
Consider a circuit composed of two independent voltage sources and resistors. We seek the equivalent Thévenin source between nodes A and M.



Step 1: Calculation of the open-circuit voltage E_{Th}

From the circuit, we observe that Millman's theorem can be directly applied. Therefore:

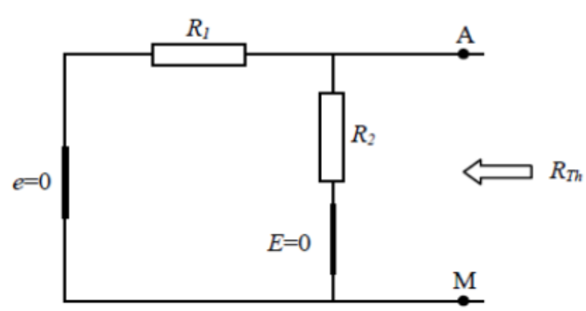
$$E_{Th} = \frac{\frac{e}{R_1} + \frac{E}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$



Step 2: Calculation of R_{Th}

To determine R_{Th}, the load R_L is removed and the independent sources e and E are short-circuited. The equivalent resistance seen from terminals A and M is then:

$$R_{Th} = R_1 \parallel R_2$$



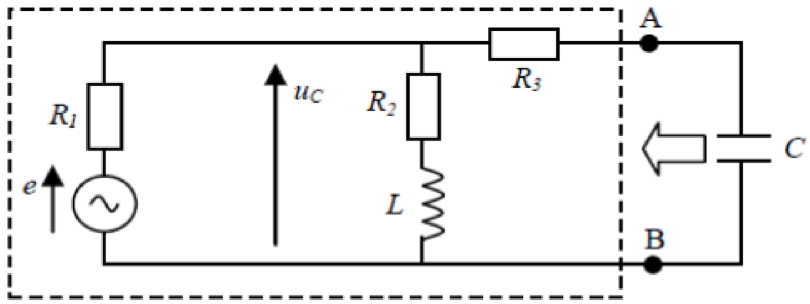
I.5. Norton's Theorem

A linear circuit placed between two terminals A and B can be replaced by an equivalent Norton generator consisting of a current source I_N in parallel with an internal resistance (or impedance) R_N.

- I_N is the short-circuit current between terminals A and B.
- R_N is calculated in the same manner as the Thévenin resistance.

Example

Determine the Norton equivalent circuit of the two-terminal network AB shown in Figure I.8.



Step 1: Calculation of the short-circuit current I_N

We observe that R_1 is in series with R_{eq}. Thus:

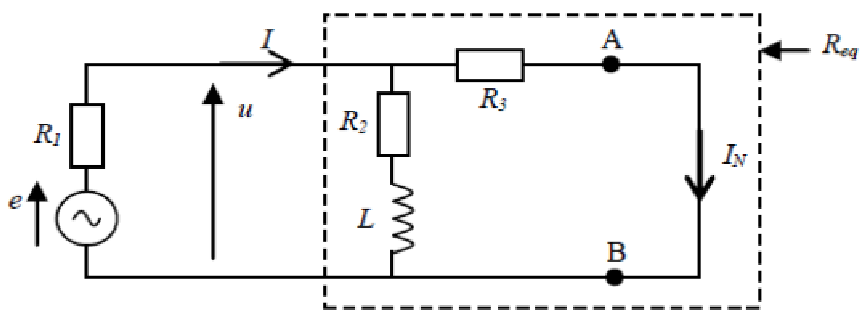
$$I = \frac{e}{R_1 + R_{eq}}$$

where:

$$R_{eq} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + j\omega L}}$$

Since R_3 is in parallel with the elements R_2 and $Z_L = j\omega L$, we apply the current divider theorem:

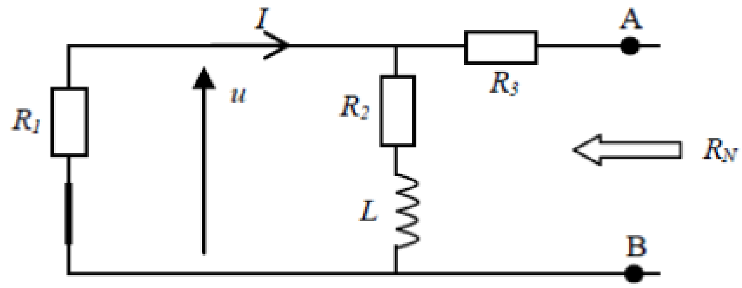
$$I_N = \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_2 + j\omega L}} I$$



Step 2: Calculation of the Norton Resistance

To determine R_N, the voltage source e is deactivated (short-circuited), and the equivalent resistance seen from terminals A and B is calculated:

$$R_N = R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + j\omega L}}$$



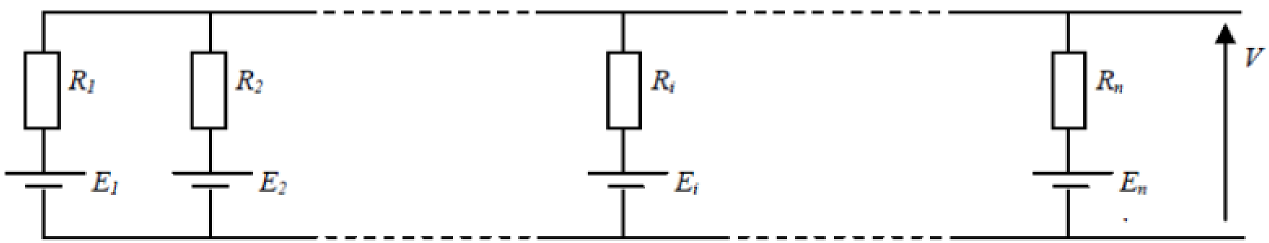
Millman's Theorem

Consider n branches connected in parallel, each containing a linear circuit that can be replaced by its Thévenin equivalent (a voltage source in series with a resistance or impedance).

Figure I.8. Application of Millman's theorem to parallel branches, each comprising a voltage source and an impedance.

The voltage V across the parallel network is given by:

$$V = \frac{\sum_{i=1}^{n} \frac{E_i}{R_i}}{\sum_{j=1}^{n} \frac{1}{R_j}}$$



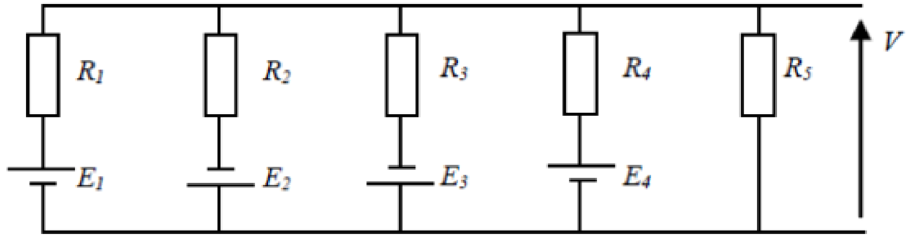
Example

Consider the electrical circuit shown in Figure I.9. We are required to calculate the voltage across the resistor R_5.

Figure I.9. Example of application of Millman's theorem.

By directly applying Millman's theorem, we obtain:

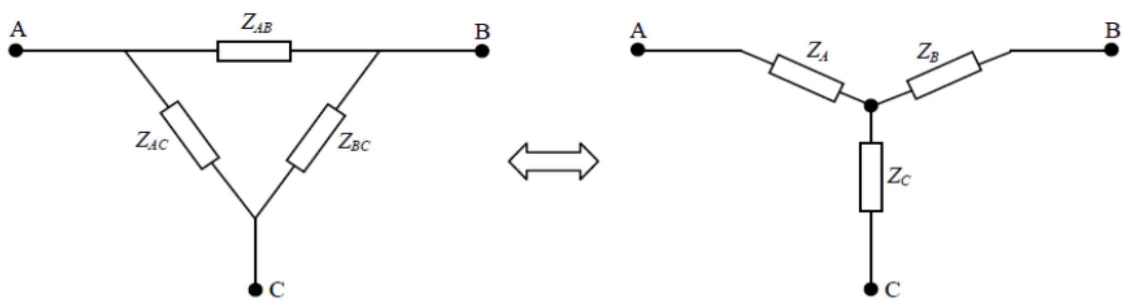
$$V = \frac{\frac{E_1}{R_1} - \frac{E_2}{R_2} - \frac{E_3}{R_3} + \frac{E_4}{R_4} + \frac{0}{R_5}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}}$$



Kennelly's Theorem

Kennelly's theorem establishes an equivalence between impedances arranged in a triangle (Δ configuration) and impedances arranged in a star (Y configuration).

Figure I.10. Left: triangle configuration. Right: star configuration.



Triangle-to-Star Conversion

The resistance (or impedance) of a branch of the equivalent star network is equal to the product of the two adjacent triangle impedances divided by the total sum of the triangle impedances.

$$Z_A = \frac{Z_{AB} Z_{AC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

$$Z_B = \frac{Z_{AB} Z_{BC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

$$Z_C = \frac{Z_{AC} Z_{BC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

Star-to-Triangle Conversion

The resistance (or impedance) of a branch of the equivalent triangle network is equal to the sum of the products of the star impedances taken two at a time, divided by the impedance of the opposite branch.

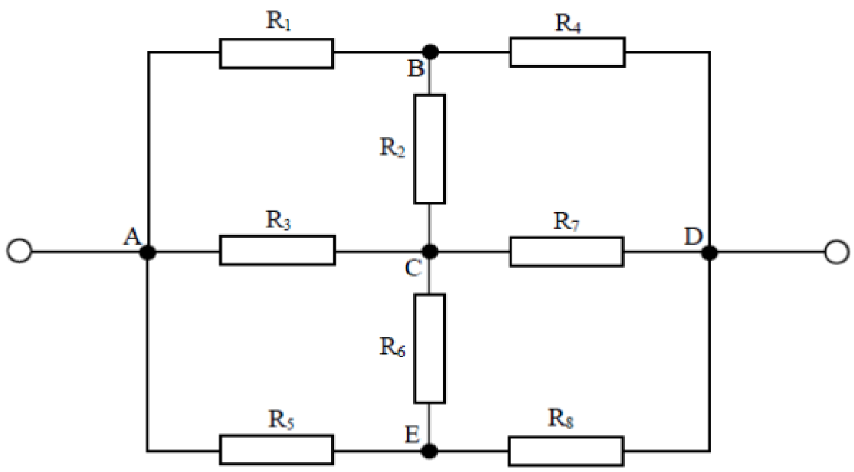
$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_C}$$

$$Z_{AC} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_B}$$

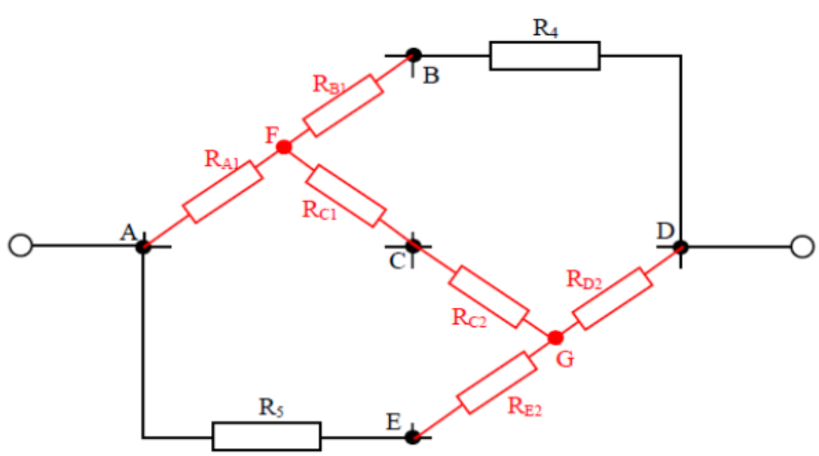
$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_A Z_C}{Z_A}$$

Example

We aim to determine the equivalent resistance of the two-terminal network shown in Figure I.11.



We begin by transforming the two triangle configurations ABC and CDE into equivalent star configurations.

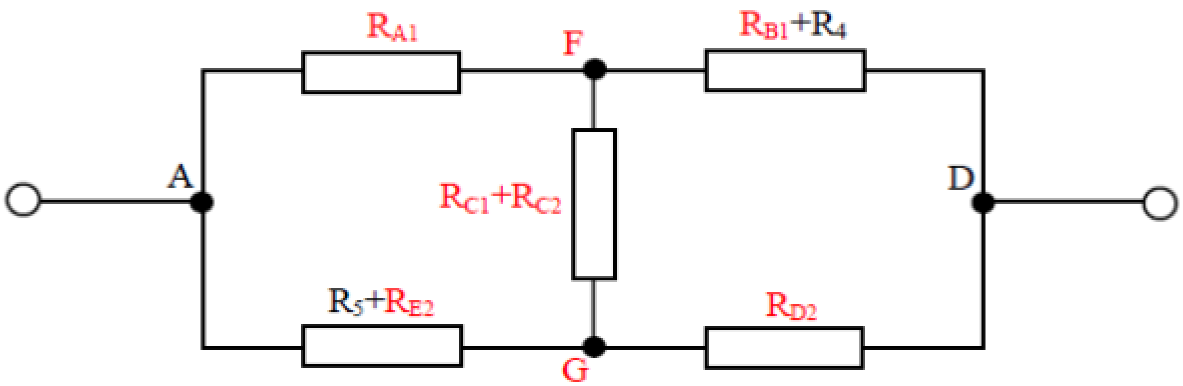


With:

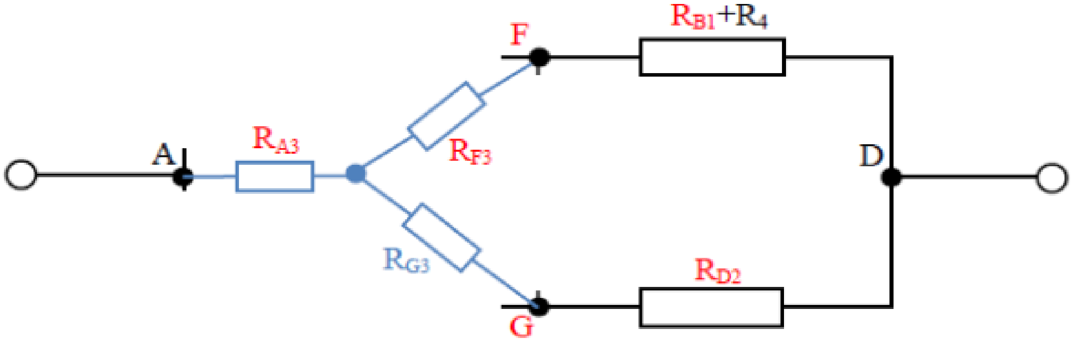
$$R_{A1} = \frac{R_1 R_3}{R_1 + R_2 + R_3}, \quad R_{B1} = \frac{R_1 R_2}{R_1 + R_2 + R_3}, \quad R_{C1} = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_{C2} = \frac{R_6 R_7}{R_6 + R_7 + R_8}, \quad R_{D2} = \frac{R_7 R_8}{R_6 + R_7 + R_8}, \quad R_{E2} = \frac{R_6 R_8}{R_6 + R_7 + R_8}$$

The network is then simplified into two triangle configurations.



In this case, a single additional triangle-to-star transformation is sufficient.



It is now clear that the equivalent resistance of the two-terminal network can be written as:

$$R_{\text{dipole}} = R_{A3} + (R_{F3} + (R_{B1} + R_4)) \parallel (R_{G3} + R_{D2})$$

with:

$$R_{A3} = \frac{R_{A1}(R_5 + R_{E2})}{R_{A1} + (R_{C1} + R_{C2}) + (R_5 + R_{E2})}$$

$$R_{F3} = \frac{R_{A1}(R_{C1} + R_{C2})}{R_{A1} + (R_{C1} + R_{C2}) + (R_5 + R_{E2})}$$

$$R_{G3} = \frac{(R_{C1} + R_{C2})(R_5 + R_{E2})}{R_{A1} + (R_{C1} + R_{C2}) + (R_5 + R_{E2})}$$