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Algebra 2 : Exercise Series N°03

Note : The (*) part is left for students.

Exercise 01 :

We define the following matrices :

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 4 \\ 2 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 3 & 0 \\ -4 & 5 & -1 \end{pmatrix},$$

$$E = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad F = \begin{pmatrix} -2 & 2 & 1 \\ 2 & 0 & -2 \\ 1 & 3 & 2 \end{pmatrix}, \quad G = \begin{pmatrix} -2 & 1 & 2 \\ 3 & 1 & 5 \\ 4 & -6 & -3 \end{pmatrix}.$$

1. Compute, if possible :

   a. $-2A$, A^2, B^2, B^2-2A, AC, CA, CD, DC, $C-E$, EB, BE, FG.

   b. $A^T, B^T, D^T, \mathrm{Tr}(A), \mathrm{Tr}(B), \mathrm{Tr}(D), \mathrm{Tr}(A+B)$.

   c. $\det(A), \det(A^T), \det(B), \det(A+B), \det(AB), \det(D), \det(F), \det(G)(*)$.

2. Verify that A and C are invertible (use two methods), and determine their inverses.

3. Verify that F is invertible, and calculate the cofactor matrix of F ($\mathrm{Cof}(F)$). Deduce the inverse matrix F^{-1}.

4. Same questions for G **(*)**.

Exercise 02 : Consider the following matrices :

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad (*)B = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}.$$

1. Calculate A^2 and A^3. Deduce the expression for A^n for all $n \geq 1$. Same questions for matrix B.

Exercise 03 :

Consider in $M_2(\mathbb{R})$ the following matrices :

$$A = \begin{pmatrix} 5 & -4 \\ 4 & -3 \end{pmatrix}, \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

1. Determine the matrix $B \in M_2(\mathbb{R})$ such that $A = I_2 + 4B$.

2. Calculate the matrix $-A^2 + 2A - I_2$.

3. Deduce whether the matrix A is invertible, and determine its inverse A^{-1} if it exists.

4. Show that $\forall n \in \mathbb{N}^*, A^n = I_2 + 4nB$.