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First year in Computer Science — 2025-2026

Algebra 2 : Exercise Series N°02

Note : The (*) part is left for students.

Exercise 01 :

Among the following mappings, which ones are linear?

1. $f_1 : \mathbb{R}^2 \to \mathbb{R}$
   $(x, y) \mapsto f_1(x, y) = x + y$
2. $f_2 : \mathbb{R}^2 \to \mathbb{R}^2$
   $(x, y) \mapsto f_2(x, y) = (x - y, 0)$
3. $f_3 : \mathbb{R}^2 \to \mathbb{R}^2$
   $f_3(x, y) = (x - y, -2x + 2y)$
4. $f_4 : \mathbb{R}^3 \to \mathbb{R}^2$
   $(x, y, z) \mapsto f_4(x, y, z) = (-2x + y + z, x - 2y + z)$
5. $f_5 : \mathbb{R}^3 \to \mathbb{R}^3$
   $(x, y, z) \mapsto f_5(x, y, z) = (-x + y, x - z, y)$
6. $f_6 : \mathbb{R}^3 \to \mathbb{R}^2$
   $f_6(x, y, z) = \big(x + \sqrt{2}y + z, 2x + y - 5z\big)$ (*)
7. $f_7 : \mathbb{R} \to \mathbb{R}^2$
   $x \mapsto f_7(x) = (x, -x)$

- Determine $f_7 \circ f_1$. Is it linear?

Exercise 02 :

Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by
$f(x, y, z) = (y + z, x + z, x)$.

1. Show that f is an endomorphism.
2. Determine $\ker(f)$ and $\mathrm{Im}(f)$. Is f injective? Surjective? An automorphism?
3. Provide the formula for defining f^{-1}.

Exercise 03 :

Let the linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by
$f(x, y, z) = (-2x + y + z, x - 2y + z)$.

1. Determine a basis for $\ker(f)$ and deduce $\mathrm{rank}(f)$ (that is $\dim \mathrm{Im}(f)$).
2. Determine a basis for $\mathrm{Im}(f)$.
3. Is f an isomorphism?

Exercise 04 :
Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by
$g(x, y) = (x - y, -2x + 2y)$.

1. Show that g is linear. (*)
2. Show that g is neither injective nor surjective.
3. Determine a basis for $\ker(g)$ and a basis for $\mathrm{Im}(g)$.

Exercise 05 : (Home-Work)
Let the mapping $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by
$f(x, y) = (x + y, x - y)$.

1. Show that f is linear.
2. Determine $\ker(f)$ and $\mathrm{Im}(f)$ and provide their dimensions. Is f bijective ?
3. Determine $f \circ f$.

Exercise 06 : (Home-Work)

We consider the linear mappings f and g defined as follows :

$$\begin{aligned} f : \mathbb{R}^3 &\to \mathbb{R}^3 \\ (x, y, z) &\mapsto f(x, y, z) = (-x + y + z, x - y, x - y + z) \end{aligned}$$

and

$$\begin{aligned} g : \mathbb{R}^2 &\to \mathbb{R}^3 \\ (x, y) &\mapsto g(x, y) = (y, x, x + y) \end{aligned}$$

1. Determine $\ker(f)$, $\mathrm{Im}(f)$, $\ker(g)$, and $\mathrm{Im}(g)$, and specify their dimensions.
2. Determine, when possible, $g \circ f$ and $f \circ g$.
3. Determine $f \circ f$.