Ex 9

$$A(0, y_A), \quad B(a, 0), \quad C(-a, 0)$$

$$G(0, y_G)$$

$$\vec{F}_G = \vec{F}_{A/G} + \vec{F}_{B/G} + \vec{F}_{C/G}$$

$$\vec{F}_G = K\frac{q\,q}{AG^2}\,\vec{u}_{AG} + K\frac{-q\,q}{BG^2}\,\vec{u}_{BG} + K\frac{q\,q}{CG^2}\,\vec{u}_{CG}$$

$$\sin 60^\circ = \frac{y_A}{2a} \Rightarrow y_A = 2a\sin 60^\circ = \sqrt{3}\,a$$

$$\tan 30^\circ = \frac{y_G}{a} \Rightarrow y_G = a\tan 30^\circ = \frac{a}{\sqrt{3}}$$

$$A(0, \sqrt{3}a), \quad B(a, 0), \quad C(-a, 0), \quad G\left(0, \frac{a}{\sqrt{3}}\right)$$

$$\vec{AG} = (0-0)\vec{i} + \left(\frac{a}{\sqrt{3}} - \sqrt{3}a\right)\vec{j}$$

$$\vec{AG} = -\frac{2a}{\sqrt{3}}\vec{j}$$

$$\vec{BG} = (0-a)\vec{i} + \left(\frac{a}{\sqrt{3}} - 0\right)\vec{j}$$

$$\vec{BG} = -a\vec{i} + \frac{a}{\sqrt{3}}\vec{j}$$

$$\vec{CG} = (0-(-a))\vec{i} + \left(\frac{a}{\sqrt{3}} - 0\right)\vec{j}$$

$$\vec{CG} = a\vec{i} + \frac{a}{\sqrt{3}}\vec{j}$$

$$\vec{F}_G = Kq^2\left(\frac{\vec{AG}}{AG^3} - \frac{\vec{BG}}{BG^3} + \frac{\vec{CG}}{CG^3}\right)$$

$$AG = \sqrt{\left(\frac{2a}{\sqrt{3}}\right)^2} = \frac{2a}{\sqrt{3}} \Rightarrow AG^3 = \frac{8a^3}{3\sqrt{3}}$$

$$BG = \sqrt{(-a)^2 + \left(\frac{a}{\sqrt{3}}\right)^2} = \sqrt{\frac{4a^2}{3}} = \frac{2a}{\sqrt{3}} \Rightarrow BG^3 = \frac{8a^3}{3\sqrt{3}}$$

$$CG = \sqrt{a^2 + \left(\frac{a}{\sqrt{3}}\right)^2} = \frac{2a}{\sqrt{3}} \Rightarrow CG^3 = \frac{8a^3}{3\sqrt{3}}$$

$$\vec{F}_G = Kq^2 \left[\frac{-\frac{2a}{\sqrt{3}}\vec{j}}{\frac{8a^3}{3\sqrt{3}}} - \frac{-a\vec{i} + \frac{a}{\sqrt{3}}\vec{j}}{\frac{8a^3}{3\sqrt{3}}} + \frac{a\vec{i} + \frac{a}{\sqrt{3}}\vec{j}}{\frac{8a^3}{3\sqrt{3}}}\right]$$

$$\vec{F}_G = Kq^2 \left[-\frac{3}{4a^2}\vec{j} - \left(-\frac{3\sqrt{3}}{8a^2}\vec{i} + \frac{3}{8a^2}\vec{j}\right) + \left(\frac{3\sqrt{3}}{8a^2}\vec{i} + \frac{3}{8a^2}\vec{j}\right)\right]$$

$$\boxed{\vec{F}_G = Kq^2 \left(\frac{3\sqrt{3}}{4a^2}\vec{i} - \frac{3}{4a^2}\vec{j}\right)}$$

$$\boxed{\vec{F}_G = \frac{3Kq^2}{4a^2}\left(\sqrt{3}\,\vec{i} - \vec{j}\right)}$$

Ex 10

Coordinate method

$$\cos 45^\circ = \frac{B_x}{a} \Rightarrow \frac{\sqrt{2}}{2} = \frac{B_x}{a} \Rightarrow B_x = \frac{\sqrt{2}}{2}a$$

$$\sin 45^\circ = \frac{B_y}{a} \Rightarrow B_y = \frac{\sqrt{2}}{2}a$$

Coordinates

$$D(0,0)$$

$$B\left(\frac{\sqrt{2}}{2}a, \frac{\sqrt{2}}{2}a\right)$$

$$C\left(-\frac{\sqrt{2}}{2}a, \frac{\sqrt{2}}{2}a\right)$$

$$A(0, \sqrt{2}a)$$

Distances

$$AD = \sqrt{2}a$$

$$BD = \sqrt{\frac{2}{4}a^2 + \frac{2}{4}a^2} = a$$

$$CD = a$$

Electric field expression

$$\vec{E}_D = \vec{E}_A + \vec{E}_B + \vec{E}_C$$

$$\vec{E}_D = k\left(\frac{-q}{AD^3}\vec{AD} + \frac{q}{BD^3}\vec{BD} + \frac{q}{CD^3}\vec{CD}\right)$$

Vectors

$$\vec{AD} = (0-0)\hat{i} + (0-\sqrt{2}a)\hat{j} = -\sqrt{2}a\,\hat{j}$$

$$\vec{BD} = \left(0-\frac{\sqrt{2}}{2}a\right)\hat{i} + \left(0-\frac{\sqrt{2}}{2}a\right)\hat{j}$$

$$\vec{CD} = \left(0+\frac{\sqrt{2}}{2}a\right)\hat{i} + \left(0-\frac{\sqrt{2}}{2}a\right)\hat{j}$$

Substitution

$$\vec{E}_D = kq\left(\frac{-(-\sqrt{2}a)\hat{j}}{(\sqrt{2}a)^3} + \frac{-\frac{\sqrt{2}}{2}a\hat{i} - \frac{\sqrt{2}}{2}a\hat{j}}{a^3} + \frac{\frac{\sqrt{2}}{2}a\hat{i} - \frac{\sqrt{2}}{2}a\hat{j}}{a^3}\right)$$

Simplification

$$\vec{E}_D = kq\left(\frac{1}{2a^2}\hat{j} - \frac{\sqrt{2}}{2a^2}\hat{i} - \frac{\sqrt{2}}{2a^2}\hat{j} + \frac{\sqrt{2}}{2a^2}\hat{i} - \frac{\sqrt{2}}{2a^2}\hat{j}\right)$$

$$\vec{E}_D = kq\left(\frac{1}{2a^2} - \frac{2\sqrt{2}}{2a^2}\right)\hat{j}$$

$$\boxed{\vec{E}_D = \frac{kq}{a^2}\left(\frac{1}{2} - \sqrt{2}\right)\hat{j}}$$

Force when charge $+2q$ is placed at D

$$\vec{E}_B + \vec{E}_C = 0$$

$$\vec{E}_0 = 2\vec{E}_A$$

$$\vec{E}_0 = \vec{E}_D + \vec{E}_A$$

$$AD = a\sqrt{2} \quad \Rightarrow \quad AO = DO = \frac{a\sqrt{2}}{2}$$

$$\vec{E}_A = \frac{k(-q)}{\left(\frac{a\sqrt{2}}{2}\right)^2}(-\hat{j})$$

$$\vec{E}_0 = \left(\frac{2kq}{\frac{1}{2}a^2} - \frac{kq}{\frac{1}{2}a^2}\right)\hat{j}$$

$$\boxed{\vec{E}_0 = \frac{2kq}{a^2}\hat{j}}$$

When the question is limited to determining the magnitude of the electric field, this solution method may be used.

Electric field at point D

$$\vec{E}_D = \vec{E}_A + \vec{E}_B + \vec{E}_C$$

$$\vec{E}_{BC} = \vec{E}_B + \vec{E}_C$$

$$\vec{E}_D = \vec{E}_A + \vec{E}_{BC}$$

Magnitudes

$$E_B = E_C = \frac{kq}{a^2}$$

Angle between $\vec{E}_B$ and $\vec{E}_C$:

$$\frac{\pi}{2}$$

$$E_{BC} = \sqrt{E_B^2 + E_C^2 + 2E_B E_C \cos\frac{\pi}{2}}$$

$$E_{BC} = \sqrt{E_B^2 + E_C^2}$$

$$E_{BC} = \sqrt{2E_B^2}$$

$$E_{BC} = \sqrt{2}\,E_B$$

$$E_{BC} = \frac{\sqrt{2}kq}{a^2}$$

Geometry

Diagonal of rhombus:

$$d = \sqrt{2}a$$

Vector relation

$$\vec{E}_D = \vec{E}_{BC} - \vec{E}_A$$

From Coulomb's law

$$E_A = \frac{kq}{2a^2}$$

$$E_D = \frac{kq}{a^2}\sqrt{2} - \frac{kq}{2a^2}$$

$$\boxed{E_D = \frac{kq}{a^2}\left(\sqrt{2} - \frac{1}{2}\right)}$$