Ministry of Higher Education and Scientific Research

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Problems & Exercises (of Chap 0)

Vectors, Coordinate systems

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PART 1: Vectors

Exercise 1

In a system of orthonormal axes, the following vectors are given:

$$\vec{V}_1 = 3\vec{i} + 4\vec{j} \text{ and } \vec{V}_2 = -\vec{i} + 2\vec{j}.$$

1- Calculate the magnitudes of the vectors $\|\vec{V}_1\|$, $\|\vec{V}_2\|$, and $\|\vec{V}_1 + \vec{V}_2\|$.
2- Determine the angle $\boldsymbol{\theta}$ between the vectors $\vec{V}_1$ and $\vec{V}_2$.
3- Determine the unit vector $\vec{u}$ carried by the vector $(\vec{V}_1 + \vec{V}_2)$.
4- Determine the angles $\boldsymbol{\alpha}, \boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ that $\vec{u}$ makes with the coordinate axes (*ox*), (*oy*), and (*oz*), respectively.
5- Calculate the components of the vector $\vec{V}_3 = \vec{V}_1 \wedge \vec{V}_2$.
6- Consider the triangle (OAB) defined by the endpoints of vectors $\vec{V}_1$ and $\vec{V}_2$: $\vec{V}_1 = \overrightarrow{OA}$ and $\vec{V}_2 = \overrightarrow{OB}$. Show that the area of the triangle (OAB) can be expressed in terms of $\|\vec{V}_1\|$, $\|\vec{V}_2\|$, and $\boldsymbol{\theta}$.
Verify that it can also be obtained using the relation: $\frac{1}{2}\|\vec{V}_1 \wedge \vec{V}_2\|$.

Exercise 2

In the three-dimensional orthonormal coordinate system $(O, \vec{i}, \vec{j}, \vec{k})$, we have three vectors:

$$\vec{U} = \vec{i} + \vec{j} + \vec{k};\ \vec{V} = 2\vec{i} - \vec{j} + 2\vec{k};\ \vec{W} = -2\vec{k}$$

1. Draw the three vectors $\vec{U}$, $\vec{V}$, and $\vec{W}$.
2. Calculate the magnitudes of $\|\vec{U}\|$, $\|\vec{V}\|$, and $\|\vec{W}\|$.
3. Determine the components of the unit vector $\vec{u}$ carried by $\vec{U}$.
4. Graphically represent the vector $(\vec{U} - \vec{V})$ and calculate its magnitude.
5. Calculate:

   a) The dot product $\vec{U} \cdot \vec{V}$.

   b) The cross product $\vec{U} \wedge \vec{V}$.

   c) The double cross product $(\vec{U} \wedge \vec{V}) \wedge \vec{W}$.

   d) The scalar triple product $(\vec{V} \wedge \vec{W}) \cdot \vec{U}$.

6. Determine the angle between $\vec{U}$ and $\vec{V}$.

Exercise 3

In an orthonormal base $(\vec{\imath}, \vec{\jmath}, \vec{k})$, the scalar function $\boldsymbol{F(x, y, z) = 3\, y\, x^2 - y^3\, z^2}$

1. Determine the partial derivatives $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}$ of $\boldsymbol{F}$.
2. Deduce the gradient of the function $\boldsymbol{F(x, y, z)}$, $\boldsymbol{(\vec{\nabla} F)}$, at the point $\boldsymbol{P\,(1, -2, -1)}$.

Exercise 4

Consider the scalar field $\boldsymbol{(x, y, z) = 3x^2y + y^2z^2}$ and the vector field given by:

$$\vec{V}\,(x, y, z) = xz^2\vec{\imath} + (2x^2 - y)\,\vec{\jmath} + yz^2\vec{k}$$

Calculate: the gradient ∇f, the divergence div($\vec{V}$), the curl rot($\vec{V}$).

PART 2: Coordinate Systems

Exercise 5

Convert the cylindrical coordinates $\boldsymbol{(\rho = 5, \theta = 2\pi/3, z = -7)}$ into Cartesian coordinates $\boldsymbol{(x, y, z)}$.

Exercise 6

In an orthonormal base $\boldsymbol{(\vec{\imath}, \vec{\jmath})}$, a point is given:

. a) $\boldsymbol{A\,(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})}$ Give the polar coordinates $\boldsymbol{(\rho, \theta)}$ of this point.

In an orthonormal base $\boldsymbol{(\vec{\imath}, \vec{\jmath}, \vec{k})}$, the points given are:

. b) $\boldsymbol{B\,(2, 2, 2)}$ Give the cylindrical $\boldsymbol{(\rho, \theta, z)}$ and spherical $\boldsymbol{(r, \theta, \varphi)}$ coordinates of this point.

. c) $\boldsymbol{C\,(\sqrt{2}, \frac{\pi}{4}, 3)}$ Give the Cartesian coordinates $\boldsymbol{(x, y, z)}$ of this point.

. d) $\boldsymbol{D\,(4, \frac{\pi}{3}, \frac{\pi}{6})}$ Give the Cartesian coordinates $\boldsymbol{(x, y, z)}$ of this point.

Exercise 7

Calculate by integration:

1. The surface area of a disk with radius **R**.
2. The lateral surface area of a cylinder with radius **R** and height **h**.
3. The volume of the cylinder.
4. The surface area of a sphere with radius **R**.
5. The volume of the sphere.