Solution

Ex 01 : 05 points

1- $dl_1 = d\rho$, $dl_2 = \rho\, d\varphi$, $dl_3 = dz$. (0,5)

$dV = \rho\, d\rho\, d\varphi\, dz$. (0,5)

$V = \int_{R_1}^{R_2} \rho\, d\rho \int_{0}^{\pi} d\varphi \int_{\frac{2}{3}h}^{h} dz$. (0,5)

$V = \left[\frac{1}{2}\rho^2\right]_{R_1}^{R_2} \left[\varphi\right]_0^{\pi} \left[z\right]_{\frac{2}{3}h}^{h}$ (0,5)

$V = \frac{1}{2}(R_2^2 - R_1^2)(\pi)(h - \frac{2}{3}h)$ (0,5)

$$\boxed{V = \frac{\pi}{6}(R_2^2 - R_1^2)h}$$ (0,5)

2- $V = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\theta}$

$\vec{V} = \vec{V}_1 + \vec{V}_2 \Rightarrow \vec{V}^2 = (\vec{V}_1 + \vec{V}_2)^2 = \vec{V}_1^2 + \vec{V}_2^2 + 2\vec{V}_1\vec{V}_2$ (0,5)

$\vec{V}^2 = \vec{V}.\vec{V} = V.V\cos(0) = V^2$. (0,25)

$\vec{V}_1^2 = \vec{V}_1.\vec{V}_1 = V_1.V_1\cos(0) = V_1^2$. (0,25)

$\vec{V}_2^2 = \vec{V}_2.\vec{V}_2 = V_2.V_2\cos(0) = V_2^2$. (0,25)

$\vec{V}_1.\vec{V}_2 = V_1.V_2\cos\theta$ (0,25)

$V^2 = V_1^2 + V_2^2 + 2V_1V_2\cos\theta$. (0,5)

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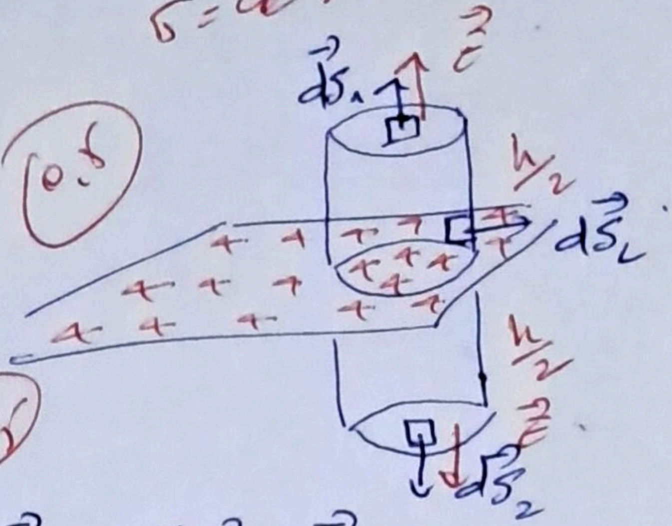
Ex 02 : 05 points

$$\Phi_T = \frac{\sum Q_{int}}{\varepsilon_0}$$ 0.5

$\sigma = ct > 0$



0.5

1) Flux :

$$\Phi_T = \iint \vec{E} \cdot \vec{dS}$$ 0,25

0,25 $$= \iint \vec{E} \cdot \vec{dS_L} + \iint \vec{E} \cdot \vec{dS_1} + \iint \vec{E} \cdot \vec{dS_2}$$

$$= \iint E \cdot dS_L \cancel{\cos \frac{\pi}{2}}^{0} + \iint E \cdot dS_1 \cos 0 + \iint \vec{E} \cdot dS_2 \cos 0$$

$$= E \iint dS_1 + E \iint dS_2 = E(S_1 + S_2)$$

We have : $S_1 = S_2 \Rightarrow \boxed{\Phi_T = 2ES}$ 0,5

2) Internal charge

$$\sigma = \frac{Q_{int}}{S} \Rightarrow \boxed{Q_{int} = \sigma S}$$ 0,5

3- Gauss's Theorem : $$\Phi_T = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$2E\cancel{S} = \frac{\sigma \cancel{S}}{\varepsilon_0} \Rightarrow E(z) = \frac{\sigma}{2\varepsilon_0}$$

$$\vec{E}(z) = \begin{cases} \frac{\sigma}{2\varepsilon_0} \vec{u}_z & z > 0 \quad 0,5 \\ -\frac{\sigma}{2\varepsilon_0} \vec{u}_z & z < 0 \quad 0,5 \end{cases}$$

(02)

4) $dV = -E_z\, dz \Rightarrow V = \int -E_z\, dz$. (0,5)

$$V(z) = \begin{cases} -\dfrac{\sigma}{2\varepsilon_0} z & z > 0 \quad (0,5) \\ \dfrac{\sigma}{2\varepsilon_0} z & z < 0 \quad (0,5) \end{cases}$$

EX 03 05 points

1) $V_0 = V_A + V_B + V_C + V_D$. (0,25)

$= K\dfrac{q_A}{a} + K\dfrac{q_B}{a} + K\dfrac{q_C}{a} + K\dfrac{q_D}{a}$. (0,25)

$= K\dfrac{-q}{a} + K\dfrac{2q}{a} + K\dfrac{3q}{a} + K\dfrac{-2q}{a}$

$= K\dfrac{q}{a}(-1+2+3-2) = 2K\dfrac{q}{a}$. (0,25)

$V_0 = 2 \cdot 9 \cdot 10^9 \cdot \dfrac{10^{-9}}{5 \cdot 10^{-2}} \Rightarrow \boxed{V_0 = 3,6 \cdot 10^2\ V}$ (0,25)

2) $\vec{E}_0 = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D$. (0,25)

(0,25)

$= K\dfrac{q_A}{a^2}\vec{u}_{AO} + K\dfrac{q_B}{a^2}\vec{u}_{BO} + K\dfrac{q_C}{a^2}\vec{u}_{CO} + K\dfrac{q_D}{a^2}\vec{u}_{DO}$

$= K\dfrac{q}{a^2}(-\vec{u}_{AO} + 2\vec{u}_{BO} + 3\vec{u}_{CO} - 2\vec{u}_{DO})$.

$\vec{u}_{AO} = \vec{i}\ ,\ \vec{u}_{BO} = -\vec{j}\ ,\ \vec{u}_{CO} = -\vec{i}\ ,\ \vec{u}_{DO} = \vec{j}$ (0,5)

$\vec{E}_O = K \dfrac{q}{a^2}\left(-\vec{i} - 2\vec{j} - 3\vec{i} - 2\vec{j}\right)$

$\boxed{\vec{E}_O = -4K \dfrac{q}{a^2}\left(\vec{i} + \vec{j}\right)}$ (0,25)

$\vec{E}_O = -4 \cdot 9 \cdot 10^{9} \cdot \dfrac{10^{-9}}{25 \cdot 10^{-4}}\left(\vec{i} + \vec{j}\right)$

$= -1{,}44 \cdot 10^{4}\left(\vec{i} + \vec{j}\right)$

$\boxed{\|\vec{E}_O\| = 2{,}03 \cdot 10^{4}\ N/C}$ (0,25)

3)- $\vec{F}_O = q' \vec{E}_O = -\dfrac{10^{-9}}{2}\left(-1{,}44 \cdot 10^{4}\left(\vec{i} + \vec{j}\right)\right)$ (0,25)

$\boxed{\vec{F}_O = 7{,}2 \cdot 10^{-6}\left(\vec{i} + \vec{j}\right)}$ (0,25)

4)- $\vec{E}_D = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_O$

$= K \dfrac{q_A}{AD^2}\vec{u}_{AD} + K \dfrac{q_B}{BD^2}\vec{u}_{BD} + K \dfrac{q_C}{CD^2}\vec{u}_{CD} + K \dfrac{q_O}{OD^2}\vec{u}_{OD}$

$\boxed{AD = \sqrt{2}a,\ CD = \sqrt{2}a,\ BD = 2a,\ OD = a}$ (0,5)

$\vec{u}_{BD} = -\vec{j}\ ,\ \vec{u}_{OD} = -\vec{j}\ ,\ \vec{u}_{AD} = \dfrac{1}{\sqrt{2}}\vec{i} - \vec{j}\ ,\ \vec{u}_{CD} = \dfrac{1}{\sqrt{2}}-\vec{i} - \vec{j}$

(0,25) (0,25) (0,25) (0,25)

$$\vec{E}_D = K\frac{q}{a^2}\left(-\frac{1}{2}\vec{u}_{AD} + \frac{2}{4}\vec{u}_{BD} + \frac{3}{2}\vec{u}_{CD} - \frac{1}{2}\vec{u}_{OD}\right)$$

$$= K\frac{q}{a^2}\left(-\frac{1}{2\sqrt{2}}\vec{i} + \frac{1}{2\sqrt{2}}\vec{j} - \frac{1}{2}\vec{j} - \frac{3}{2\sqrt{2}}\vec{i} - \frac{3}{2\sqrt{2}}\vec{j} + \frac{1}{2}\vec{j}\right).$$

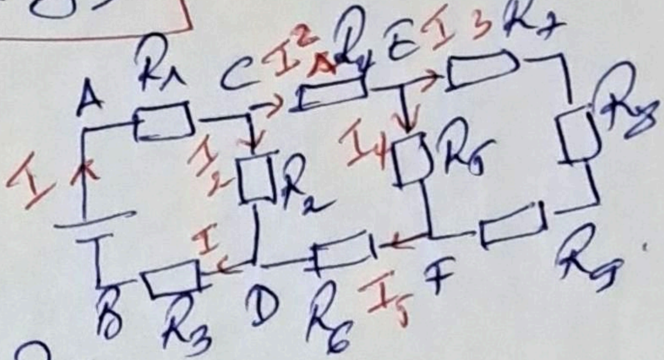
$$= K\frac{q}{a^2}\left(-\frac{2}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right) = -K\frac{q}{\sqrt{2}a^2}\left(2\vec{i} + \vec{j}\right).$$

$$= -9.10^9\,\frac{10^{-9}}{\sqrt{2}\,25.10^{-4}}\left(2\vec{i} + \vec{j}\right).$$

$$\boxed{\vec{E}_D = -2{,}54.10^3\left(2\vec{i} + \vec{j}\right)}$$

(0,5)

EX04 05 points



$$R_{789} = R_7 + R_8 + R_9$$

$$= 4 + 4 + 16 = 24\ \Omega.$$

(0,25)

$$\frac{1}{R_{5789}} = \frac{1}{R_5} + \frac{1}{R_{789}} = \frac{1}{24} + \frac{1}{24}$$

$$R_{5789} = 12\ \Omega.$$

(0,25)

$$R_{456789} = R_4 + R_{5789} + R_6$$

$$= 12 + 6 + 6 = 24\ \Omega.$$

(0,25)

(05)

$$\frac{1}{R_{2456789}} = \frac{1}{R_2} + \frac{1}{R_{456789}} = \frac{1}{24} + \frac{1}{24} = \frac{1}{12}$$

$$R_{2456789} = 12\ \Omega$$ (0,25)

$$R_{eq} = R_{AB} = R_1 + R_{2456789} + R_3$$
$$= 8 + 12 + 8$$

$$\boxed{R_{AB} = 28\ \Omega}$$ (0,5)

2) $E = R_{AB} I \Rightarrow I = \frac{E}{R_{AB}} = \frac{56}{28} \Rightarrow \boxed{I = 2A}$ (0,25) (0,25)

3) $V_{AC} = R_{AC} I = 8 . 2 = 16V$ $\boxed{V_{AC} = 16V}$ (0,5)

Loop ACDB. $E - V_1 - V_2 - V_3 = 0$

$E - R_1 I - R_2 I_2 - R_3 I = 0$ (0,5)

$E - I(R_1 + R_3) - R_2 I_2 = 0$

$$I_2 = \frac{E - I(R_1 + R_3)}{R_2} = \frac{56 - 2(8+8)}{24}$$

$I_{CD} = \boxed{I_2 = 1A}$ (0,5)

4)- Loop CEDF $R_2 I_2 - R_4 I_1 - R_5 I_4 - R_6 I_5 = 0$

(06)

$I = I_1 + \frac{I}{2}$, $I = I_5 + \frac{I}{2}$ $\Rightarrow I_1 = I_5$.

$I_1 = I_5 = I - \frac{I}{2} = 2 - 1 = 1\ A$

0,5 $\boxed{I_1 = I_5 = 1\ A}$ $\Rightarrow I_{CE} = I_{FD} = 1\ A$.

$V_{EF} = V_{DC} - V_{CE} - V_{FD}$.

$= R_2 I_2 - R_4 I_1 - R_6 I_5 = 24 \cdot 1 - 6 \cdot 1 - 6 \cdot 1$

$\boxed{V_{EF} = 12\ V}$ 0,5

$V_{EF} = R_5 I_4$ (ou I_{EF}) $\Rightarrow I_{EF} = \frac{V_{EF}}{R_5}$.

$I_{EF} = \frac{12}{24} = 0{,}5\ A$ $\boxed{I_{EF} = 0{,}5\ A}$

0,5