Chapter 3: Electrokinetics

1. Introduction

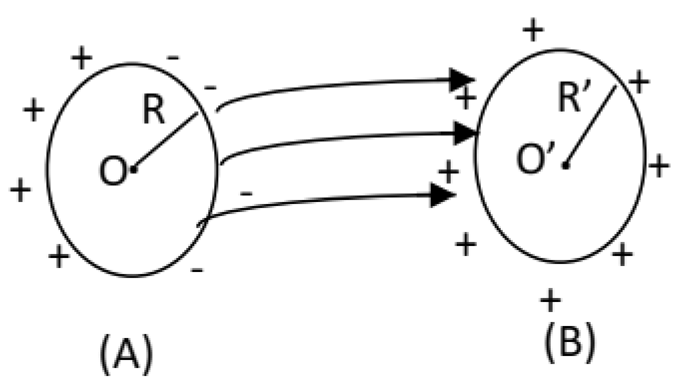
Electrokinetics is the study of the motion of electric charges. In other words, it is the study of electrical circuits and, more broadly, the movement of electricity within conducting materials, as opposed to electrostatics, which examines phenomena and laws related to stationary electric charges.

A conductor is a medium in which charges are able to move freely. Examples include metals, ionic solutions, and semiconductors. In metals, only electrons are able to move freely. In other media, different charge carriers may be in motion—for instance, in electrolytes there are multiple positive and negative ions, and in ionized gases various charged particles move as well.

An electric current is defined as the collective movement of electric charges within a conducting medium.

1.1. Origin of Electric Current

Consider two conductors A and B in electrostatic equilibrium under partial influence:



Equilibrium state

If the two conductors are connected by a conducting wire, they form a single conductor subjected to an electric field. In this case, charges will move from one conductor to the other through the wire. This flow of charges constitutes **an electric current**.

Note: This current is temporary; it stops once equilibrium is established. To obtain a continuous current, a generator is required.

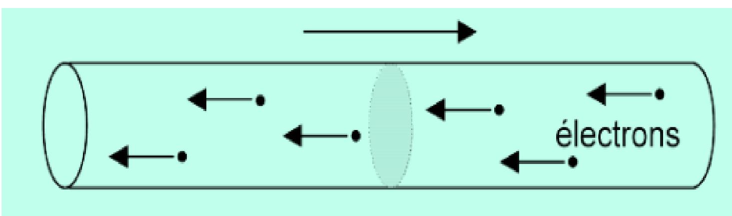
Definition: An electric current is the collective motion of electric charge carriers. In metals, these carriers are electrons (negative charges), while in liquids and gases the carriers may be electrons or positive or negative ions.

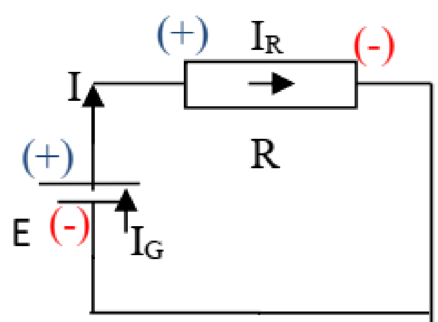
The elementary charge is that of the electron:

$Q = e = -1.6\times10^{-19}$ coulombs (C).

1.2. The Conventional Direction of Electric Current

The conventional direction of current, established by Ampère, is opposite to the actual motion of electrons. In fact, current is considered to flow from the positive terminal to the negative terminal of an electrical generator.





Conventional Direction

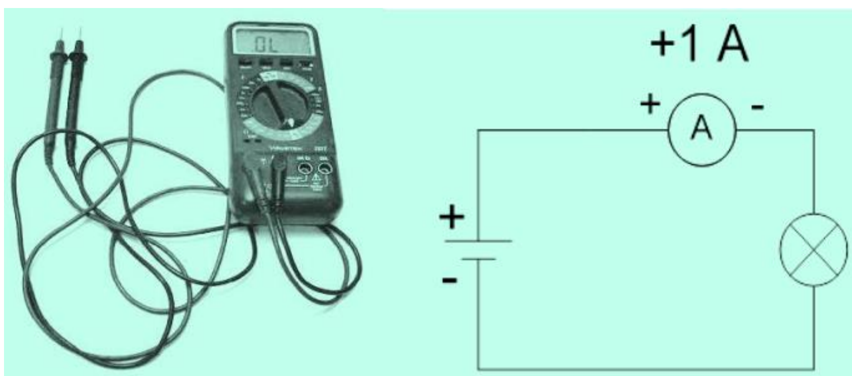
1.3. Electric Current Intensity

The intensity of an electric current measures the algebraic quantity of electricity (charge carriers) passing through the cross-section of a conductor per unit time:

$$I = \frac{dq}{dt}$$

The unit of current is the ampere **(A).**

The instrument used to measure electric current is the ammeter, which is connected in series within the circuit.



1.4. Electric Current Density

The current density, denoted by $\vec{j}$, represents the amount of charge passing through a unit area per unit time. It is a **vector quantity** that characterizes the motion of a collection of mobile charges with an average velocity $\vec{v}$ and a charge volumetric density ρ:

$$\vec{j} = \rho\,\vec{v} \text{ with } \rho = n.q = n.e$$

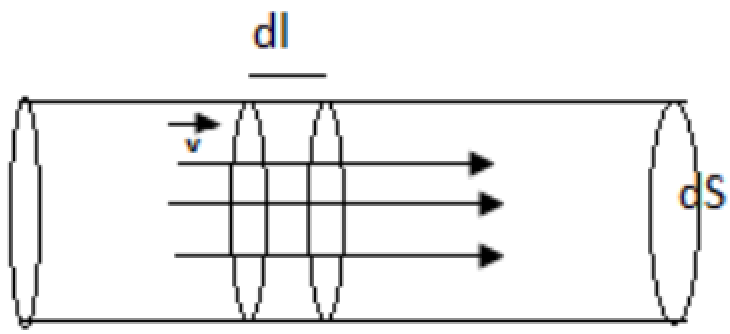
Where:

- n is the number of electrons,
- $\vec{v}$ is the average velocity of charge carriers.

Thus, for electrons in metals, we can write:

$$\vec{j} = -\, n.e.\, \vec{v}$$

In a conductor with cross-sectional area S, we have: ***dl* = *dx***



$$\overrightarrow{v_{cod}} = \frac{\overrightarrow{dx}}{dt} \text{ avec } \overrightarrow{dx} = \vec{v}dt$$

$$\begin{cases} dq = \rho dv = \rho \overrightarrow{ds}\, \overrightarrow{dx} = \rho \overrightarrow{ds}\, \vec{v}dt \\ \vec{j} = \rho \vec{v} \end{cases} \quad \text{donc } dq = \vec{j}\overrightarrow{ds}\, dt \;\; \text{alors } \boldsymbol{I = \frac{dq}{dt} = \iint \vec{j}\overrightarrow{ds}}$$

In a cylindrical conductor, the current is given by: $I = \iint \vec{j}\overrightarrow{ds}$

Definition: The electric current intensity represents the **flux of the current density vector $\vec{j}$** through a cross-section **S** of a conductor. Its unit is **A/m²**.

In general, for a wire, the relationship between current intensity and current density is: $\boldsymbol{j = I/s}$

Reminder: Voltage is the **potential difference** between two points A and B (denoted U_{AB}): $U_{AB} = V_A - V_B$.

The unit of ***U*** is the **volt (V)**.

2. Ohm's Law

2.1. Electrical Conductivity

In the presence of an electric field $\vec{E}$, there is a current density through a conductor of length ***l*** and cross-sectional area ***S***, given by:

$$\vec{j} = \sigma\vec{E} \quad \textit{With} \quad \sigma = \frac{n\,\tau\,e^2}{m_e}$$

Where:

- σ is the **electrical conductivity**,
- τ is the **average time between successive collisions** of charge carriers,
- ***n*** is the **number of charges per unit volume**,
- m_e is the **electron mass**.

Note: Electrical conductivity is expressed in **Siemens per meter (S/m)**.

2.2. Electrical Resistivity

Electrical resistivity is the **inverse of electrical conductivity**: $= \frac{1}{\sigma} = \frac{R.S}{l}$

Where ***R*** is the resistance of the conductor, ***S*** its cross-sectional area, and ***l*** its length.

Resistivity is expressed in **Ω·m**.

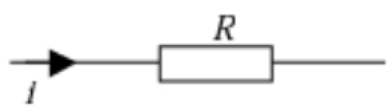
2.3. Electrical Resistance

a. Definition

The ratio between the potential difference ***U*** (voltage) across two points A and B of a metallic conductor and the current III flowing through it is constant at a fixed temperature. This ratio is denoted by ***R*** and defines the **electrical resistance**:

$$\boldsymbol{R = \frac{U}{I}}$$

The unit of resistance is the **ohm (Ω)**. Its symbol in an electrical circuit is:



b. Calculation of the Resistance of a Cylindrical Conductor

Consider a wire of length ***l*** and cross-sectional area ***S*** carrying a current of intensity ***I***. The voltage across the conductor is expressed as:

$$U = \Delta v = v_A - v_B = RI$$

The electric field along the conductor is:

$$E = -\frac{dv}{dl} \Rightarrow \int_{v_A}^{v_B} dv = -\int E dl \qquad \text{donc } v_B - v_A = -\int E dl \Rightarrow U = v_A - v_B = \int E dl$$

$$I = \iint \vec{j}\,\overrightarrow{ds} = js \text{ avec } j = \sigma E$$

$$R = \frac{U}{I} = \frac{\int E dl}{js} = \frac{E\int_A^B dl}{\sigma E s} = \frac{El}{\sigma E s} \; donc \; R = \frac{l}{\sigma s}$$

$$\begin{cases} R = \dfrac{l}{\sigma s} \\ \rho = \dfrac{1}{\sigma} \end{cases} \; donc \;\; R = \rho \frac{l}{s}$$

c. Power Calculation

The electrical power is expressed as:

$$P = U\,I = R\,I^2$$

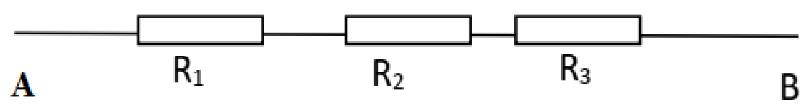
Its unit is the **watt (W)**.

This power represents the **power consumed** when the device is a **load** and the **power supplied** when the device is a **generator**.

2.4. Grouping of Resistances

There are two main ways to connect resistances in an electrical circuit: **series connection** and **parallel connection**.

a. Series Connection of Resistances



Consider three resistances connected in series. The current through each resistor is the same:

$$I_{AB} = I_{R1} = I_{R2} = I_{R3} = I \text{ With } U = R\,I$$

$$U_{AB} = U_{R1} + U_{R2} + U_{R3} \Rightarrow R_{eq} I = R_1 I_1 + R_2 I_2 + R_3 I_3 = R_1 I + R_2 I + R_3 I$$

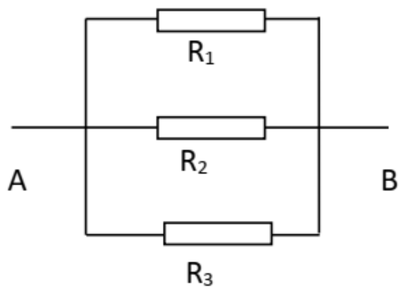
Therefore, the **equivalent resistance** for resistances in series is:

$$R_{eq} = R_1 + R_2 + R_3$$

For ***n*** resistances in series:

$$R_{eq} = \sum_{i=1}^{n} R_i$$

b. Parallel Connection of Resistances



Consider three resistances connected in parallel. The total current is the sum of the currents through each resistor:

$$I_{AB} = I_{R1} + I_{R2} + I_{R3} \; With \;\; \mathrm{I} = \mathrm{U}/\,\mathrm{R}$$

$$U_{AB} = U_{R1} = U_{R2} = U_{R3} \Rightarrow \frac{U_{AB}}{R_{eq}} = \frac{U_1}{R_1} + \frac{U_2}{R_2} + \frac{U_3}{R_3}$$

Thus, the **equivalent resistance** for resistances in parallel is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

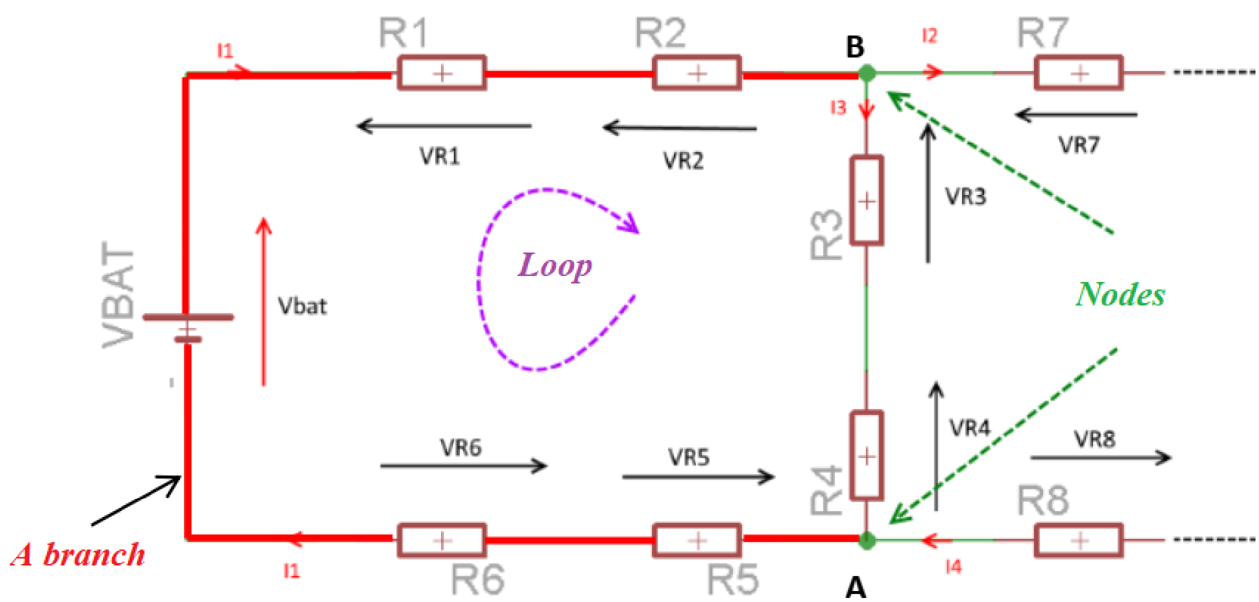
For ***n*** resistances in parallel:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

3. Electrical Network

An electrical circuit or electrical network is defined as a set of electrical components connected together by perfectly conducting wires. In a circuit, we distinguish several types of electrical devices such as resistors, generators, and capacitors.

A **node** is a point in the circuit where three or more components are connected. A **branch** of the network is the portion of the circuit located between two nodes. A **loop** (or mesh) is a closed path in the circuit formed by branches, passing through any given node **at most once**.



We can classify electrical components into two main categories:

Generator

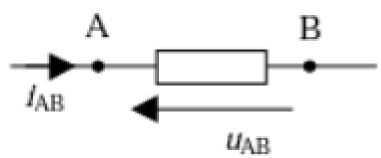
A generator is a device that produces electric current; in this element, the conventional current leaves through the positive terminal and enters through the negative terminal. The conventional direction of the potential difference UUU (voltage) for a generator is from the negative terminal to the positive terminal.

Receiver

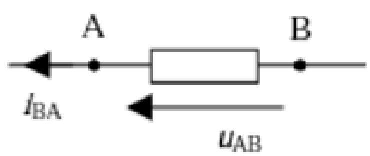
In a receiver, the current enters through the positive terminal and exits through the negative terminal.

Remark:

- In a **receiver**, the current and the voltage have **opposite directions**.



- In a **generator**, the current and the voltage have the **same direction**.



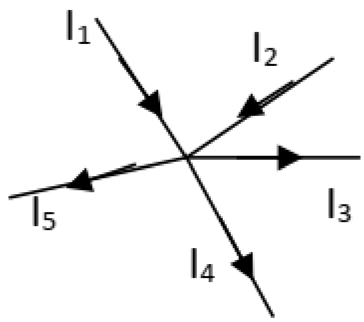
4. Kirchhoff's Laws

Kirchhoff’s two laws allow the analysis of electrical networks.

4.1. Kirchhoff’s Current Law (KCL) — Node Law

A node is a junction point where several electrical conductors meet. The sum of the currents entering a node is equal to the sum of the currents leaving the node:

$$\sum I_{entering} = \sum I_{leaving}$$

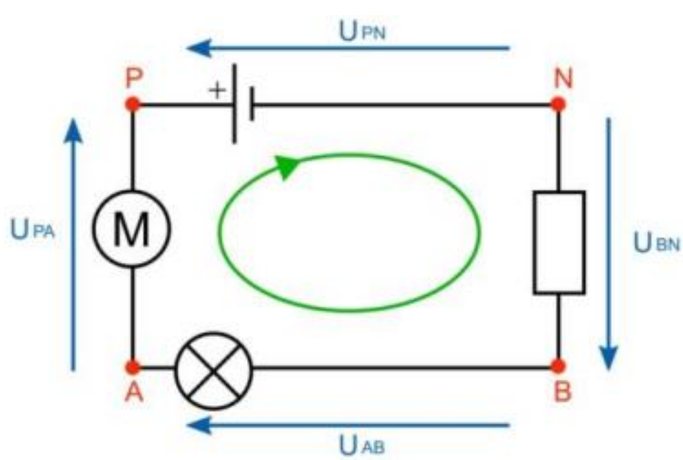


$$I_1 + I_2 = I_3 + I_4 + I_5$$

4.2. Loop law (Kirchhoff's Voltage Law)

The algebraic sum of potential differences in a loop is equal to zero:

$$\sum U_{loop} = 0$$



$$U_{PA} + U_{AB+} U_{BN+} U_{PN} = 0$$

Application example:

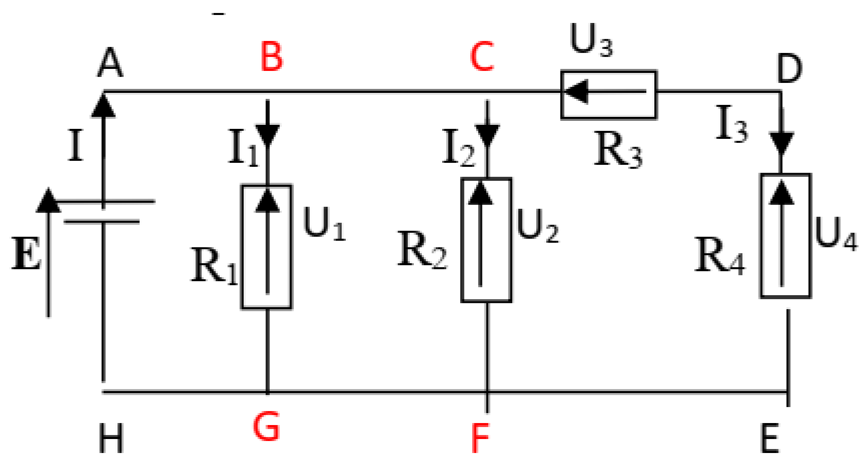
Consider the circuit shown below.

1. Calculate the value of the current ***I*** using the two Kirchhoff laws.
2. Find again the value of ***I*** using the equivalent resistance of the circuit.

Given:

$E = 24V$; $R_1 = R_2 = 20\Omega$; $R_3 = R4 = 5\Omega$.

1- In this setup we have:



4 nodes, which are: **B, C, F, G**

6 loops: **ABGHA** ; **BCFGB** ; **CDEFC** ; **ACFHA** ; **BDEGB** ; **ADEHA**

Node law:

- At point B: $I=I_1+I'$
- At point C: $I'=I_2+I_3$

Loop law (in the direction of the loops):

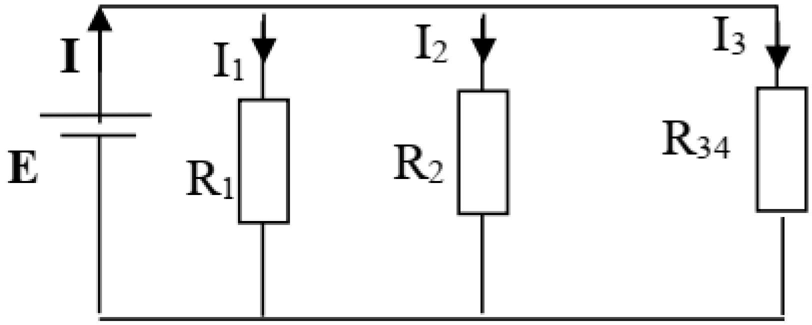
ABGHA : $E - R_1 I_1 = 0 \Rightarrow 24 - 20I_1 = 0$, $I_1 = 1{,}2\ A$

BCFGB : $R_1 I_1 - R_2 I_2 = 0 \Rightarrow 20I_1 - 20I_2 = 0$, $I_1 = I_2 = 1{,}2\ A$

CDEFC $R_2 I_2 - R_3 I_3 - R_4 I_3 = 0 \Rightarrow 20(1{,}2) - 5I_3 - 5I_3 = 0$, $I_3 = 2{,}4\ A$

$I=1{,}2+1{,}2+2{,}4=4{,}8\ A$

2. The current can be calculated by another method:



R_3 is in series with **R_4**, so:

$R_{34} = R_3 + R_4 = 5 + 5 = 10\ \Omega$

R_1, **R_2**, and **R_{34}** are connected in parallel, so:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{34}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{4}{20} \Rightarrow R_{eq} = 5\Omega$$

$$E = R_{eq} I \Rightarrow I = \frac{E}{R_{eq}} = \frac{24}{5} = 4{,}8\text{A}$$