Gauss's Theorem:

Gauss's theorem expresses the relationship between the electric flux through a closed surface (S) and the number of charges present within the volume enclosed by that surface.

It is demonstrated in this case that:

$$\boxed{\Phi(\vec{E}/S) = \oiint_{S\ fermée} \vec{E}\,\vec{dS} = \frac{1}{\varepsilon_0}\sum q_{int\,érieures}}$$

q_i :the number of charges present within the volume enclosed by (regardless of their signs).

S: is called the Gaussian surface.

Statement of the theorem:

The flux of an electric field through a closed surface is equal to the algebraic sum of the charges located inside the volume bounded by this surface, divided by the permittivity of vacuum ε_0.

Gauss's theorem is a powerful tool for calculating electrostatic fields $\vec{E}$.

In situations of symmetry, Gauss's theorem allows for simpler calculation of $\vec{E}$

$$\oiint \vec{E}\,.\,d\vec{S} = \sum_{i=1}^{n} \frac{Q_i}{\varepsilon_0}$$

Conductors, Capacitors

Definition of a conductor:

A conductor is a material in which charges move when an electrostatic force is applied to them. In metals, only electrons are mobile. The network of positive charges has only low mobility and can be considered fixed. In liquids and gases, ions also move.

Conductor in equilibrium:

Firstly, let's recall that a conductor is a body within which charges can move under the action of even a very weak electric field.

Definition:

A conductor is said to be in electrostatic equilibrium if the charges it contains are at rest.

Definition of a conductor in equilibrium:

A conductor is said to be in equilibrium if all its free charges are stationary.

Properties of a conductor in equilibrium:

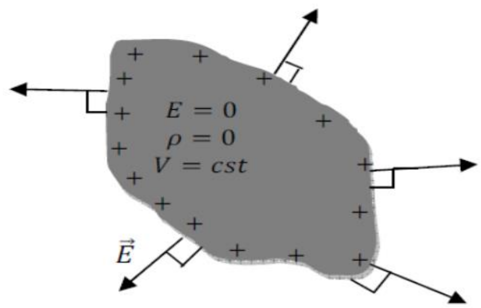
1. The electrostatic field inside the conductor is zero $\vec{E} = \vec{0}$ because the charges do not move, and $\overrightarrow{F_{ext}} = \vec{0}$
2. The potential inside the conductor is constant; it is an equipotential volume because the field is zero, and the potential follows from the relation:

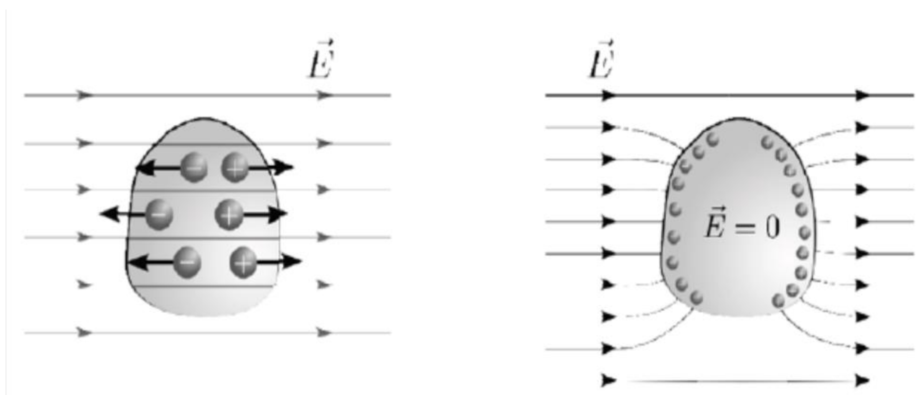
$$\vec{E} = -\overrightarrow{grad}\, V = 0 \implies V = Cte$$

3. The distribution of electric charges can only be on the surface.
   Consider a charged conductor in equilibrium. Applying Gauss's theorem at a point M on the conductor:

$$div\vec{E} = \frac{\rho}{\varepsilon_0} \text{, since } \vec{E} = \vec{0} \implies \rho = 0$$

   Therefore, the charge of the conductor can only be on the surface, with a surface density σ.



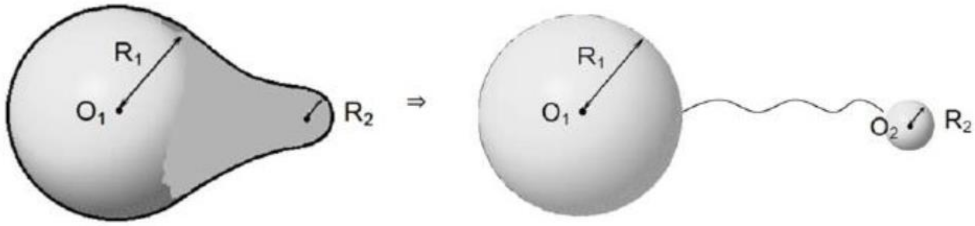


Point Effect:

This expression describes the experimental fact that, near a pointed object, the electrostatic field is always very intense. According to Coulomb's theorem, this means that the surface charge density is very high in the vicinity of a point.

We will show that near a point, the electric field is very intense.

Consider two conducting spheres of respective radii R_1 and R_2 brought to the same potential (connected by a conducting wire). Both spheres have a uniform charge density σ_1 and σ_2.



$$V(O_1) = V_1 = \frac{1}{4\pi\varepsilon_0}\iint \frac{\sigma_1\, ds}{R_1} \qquad V(O_2) = V_2 = \frac{1}{4\pi\varepsilon_0}\iint \frac{\sigma_2\, ds}{R_2}$$

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{\sigma_1 4\pi {R_1}^2}{R_1} = \frac{\sigma_1 R_1}{\varepsilon_0} \qquad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{\sigma_2 4\pi {R_2}^2}{R_2} = \frac{\sigma_2 R_2}{\varepsilon_0}$$

Since the potentials are equal:

$$\frac{\sigma_1 R_1}{\varepsilon_0} = \frac{\sigma_2 R_2}{\varepsilon_0} \Rightarrow \sigma_1 R_1 = \sigma_2 R_2 \Rightarrow \boldsymbol{\frac{\sigma_2}{\sigma_1} = \frac{R_1}{R_2}}$$

If $\boldsymbol{R1 >> R2}$, then $\boldsymbol{\sigma 2 >> \sigma 1}$

At equal potential, the charge density of a charged conductor is higher on the surface with a strong curvature (small radius) than on the surface with a weak curvature (large radius).

Capacitance of a conductor:

Consider an isolated conductor in electrostatic equilibrium, placed at a point $\boldsymbol{O}$ in space and carrying a charge $\boldsymbol{Q}$, distributed on its external surface with a surface density $\boldsymbol{\sigma}$ such that:

$$\boldsymbol{Q = \iint \sigma\, ds}$$

If the charge $\boldsymbol{Q}$ increases, the surface density $\boldsymbol{\sigma}$ increases proportionally.

This is due to the linearity of the equations governing the problem of conductor equilibrium.

The potential created by $\boldsymbol{Q}$ at a point $\boldsymbol{M}$ in space such that $\boldsymbol{OM = r}$ is given by:

$$V = K \iint \frac{\boldsymbol{\sigma\, ds}}{r}$$

This result remains valid for any point on the surface of the conductor. The integral depends solely on the geometry and dimensions of the conductor.

From this, we deduce that the ratio between the charge and the potential to which the conductor is brought only depends on the geometry of the conductor, and it is called the self-capacitance of the conductor. This is given by the expression:

$$Q = C.V \qquad C = \frac{Q}{V}$$

The capacitance C is a positive quantity, and its unit is called the Farad (F). The farad defined as the capacitance of an isolated conductor whose potential is 1 volt when it receives a charge of 1 coulomb.

The farad is a very large unit; instead, we often use its submultiples:

- Microfarad (μF) $1\mu F = 10^{-6} F$
- Nanofarad (nF) $1nF = 10^{-9} F$
- Picofarad (pF) $1\,pF = 10^{-12} F$.

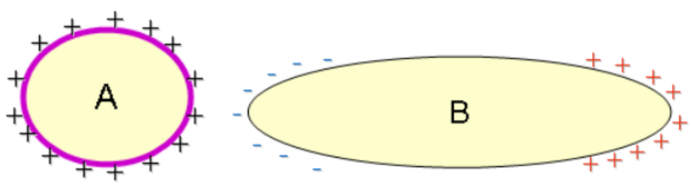
Electrostatic influence phenomenon:

a- Influence experienced by an isolated conductor:

Let:

A: Charged conductor, its charge is $Q_A > 0$

B: Neutral conductor, its charge is $Q_B = 0;\ V_B = 0\ et\ E_B = 0$



B is influenced by A.

Appearance of charges **(-)** on the part of **B** close to **A**.

Appearance of charges **(+)** on the part of **B** far from **A**.

The charge is conserved, but the surface distribution is altered.

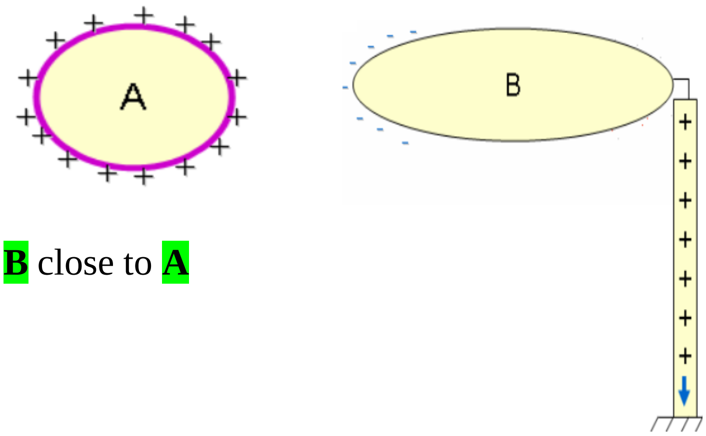
b- Influence on a conductor maintained at a constant potential:

Let:

A: Charged conductor, its charge is $Q_A > 0$

B: Neutral conductor, its charge is $Q_B = 0;\ V_B = 0$

And, If we approach **A** *to* **B** and connect **B** to the ground:



Appearance of **(-)** charges on the part of **B** close to **A**

The **(+)** charges of **B** go into the ground.

$V_B = V(terre) = 0$ But $Q_B \neq 0$

The influence phenomenon does not change the potential of the conductor but alters its total charge and the distribution of this charge.

Total influence

Total influence occurs when the influencing conductor surrounds the influenced conductor.

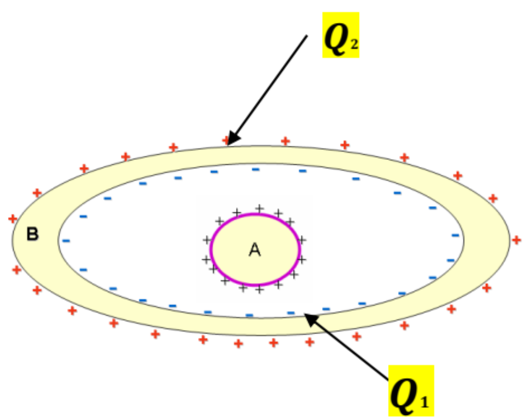
We put **A** into **B**.

For ***A*** : $Q_A > 0$

For ***B*** : $\boldsymbol{Q_B = Q_1 + Q_2}$

With $\boldsymbol{Q_1 = -Q_A}$

$\boldsymbol{Q_2 = +Q_A}$



$$\boldsymbol{Q_B = (+Q_1) + (Q_A) = 0}$$

Capacitors:

A capacitor is called a set of two conductors in total influence, one of which is hollow and completely surrounds the other; these two conductors are called the plates of the capacitor; the space separating the two plates can be empty or filled with a dielectric.

We will mainly consider, in this paragraph, capacitors in a vacuum.

Definition:

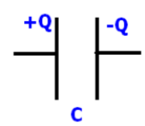
A capacitor is any system of two conductors in electrostatic influence. There are two types of capacitors:

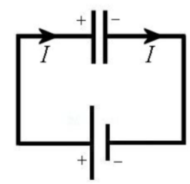
- *Capacitors with closely spaced plates.*
- *Capacitors with total influence.*

The charge of the capacitor is called the charge Q of its internal plate. Let $V1$ and $V2$ be the respective potentials of the internal and external plates.

$$\boxed{C = \frac{Q}{V}}$$

Capacitors are symbolized by:





Types of capacitors:

Capacitors can be classified into several categories based on the shape of their plates.

Flat capacitors, these plates are two flat metal plates separated by an insulator.

In a ***cylindrical capacitor***, the two plates have a cylindrical shape.

Finally, in a ***spherical capacitor***, the plates are metal spheres.

Capacitor grouping:

Capacitors can be grouped either in series or in parallel.

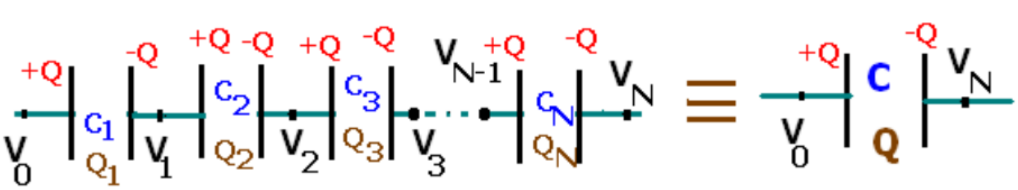
Capacitor grouping:

Capacitors can be grouped either in series or in parallel.

1. Series grouping:

Let's refer to the figure where the notations are precise; due to the total influence phenomenon caused by conductor **A**, all capacitors store the same charge Q. The voltage between the ends of the entire assembly is equal to the sum of the voltages:

The potential difference between O and N:



$$V = V_0 - V_N = (V_0 - V_1) + (V_1 - V_2) + (V_2 - V_3) + \cdots + (V_{N-1} - V_N)$$
$$\Rightarrow V = \left(\frac{Q_1}{C_1}\right) + \left(\frac{Q_2}{C_2}\right) + \left(\frac{Q_3}{C_3}\right) + \cdots + \left(\frac{Q_N}{C_N}\right) = \frac{Q}{C}$$

Since the charges are all equal (electrostatic influence), we will have:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\cdots + \frac{1}{C_N}$$

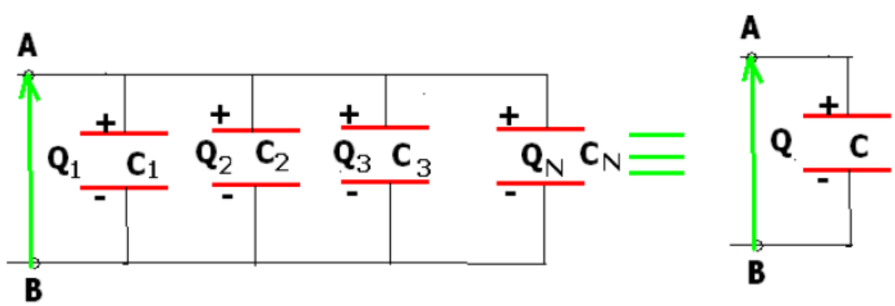
So the equivalent capacitance is:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

2. ***Parallel grouping:***

   Let's refer this time to the figure. All capacitors are subjected to the same voltage. The total charge is equal to the sum of the charges, and since the charge Q_i of each capacitor is proportional to its capacitance C_i, we can write:
   The total charge between points **A** and **B** is equal to the sum of all the charges:

$$Q = Q_1 + Q_2 + Q_3 + \cdots\cdots\cdots + Q_N$$



$$\Rightarrow (V_A - V_B)C = (V_A - V_B)\,C_1 + (V_A - V_B)C_2 + (V_A - V_B)C_3 + \cdots + (V_A - V_B)C_N$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots\cdots + C_n$$

Result: ***the equivalent capacitance is equal to the sum of the capacitances of the capacitors connected in parallel:***

$$C_{eq} = \sum_{i=1}^{n} C_i$$

Capacitors in series	Capacitors in parallel
Same charge: $$Q_1 = Q_2$$	Different charges: $$Q = Q_1 + Q_2$$
Different voltage: $$V = V_1 + V_2 + V_3 + \cdots\cdots + V_n$$ Equivalent capacitance: $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\cdots + \frac{1}{C_n}$$	Same voltage: $$V = V_1 = V_2 = V_3 = \cdots\cdots = V_n$$ Equivalent capacitance: $$C_{eq} = C_1 + C_2 + C_3 + \cdots\cdots + C_n$$