The dipole moment of the charge doublet is defined as the product of the positive charge q by the vector BA joining the negative charge to the positive charge, that is:

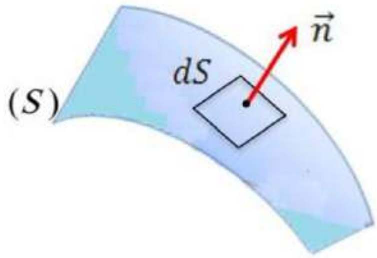
$$\vec{p} = q.\vec{d}$$

$$p = 2a.q$$

<u>Gauss's theorem (1777-1855)</u>

a) Representation of a surface by a vector

Consider a surface element and its normal vector

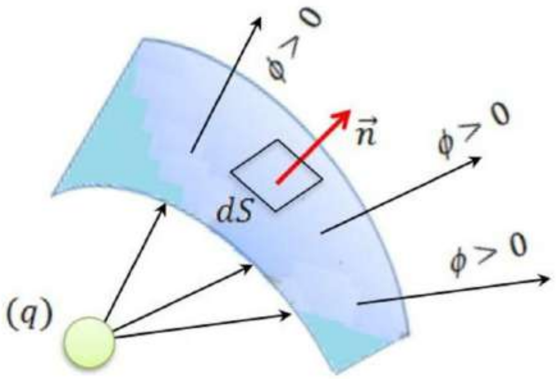


$$\boxed{\overrightarrow{dS} = \vec{n}.dS}$$

The electric field flux:

The quantity called electric field flux through a surface is.

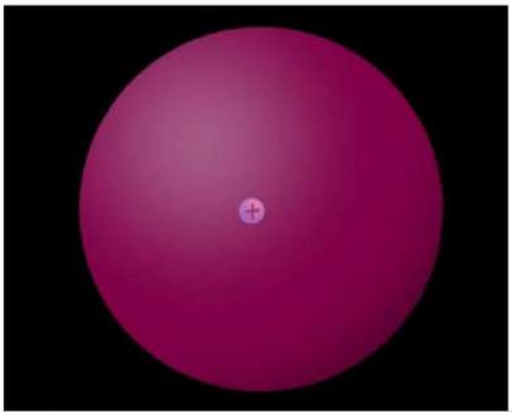
$$d\phi = \vec{E}.\overrightarrow{dS} = \vec{E}.\vec{n}.dS \qquad \phi = \iint_S \vec{E}.\vec{n}.dS$$

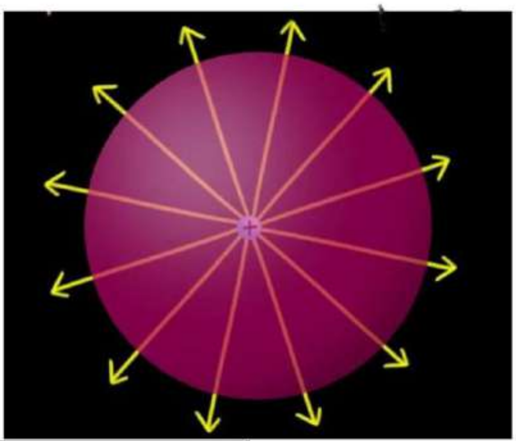


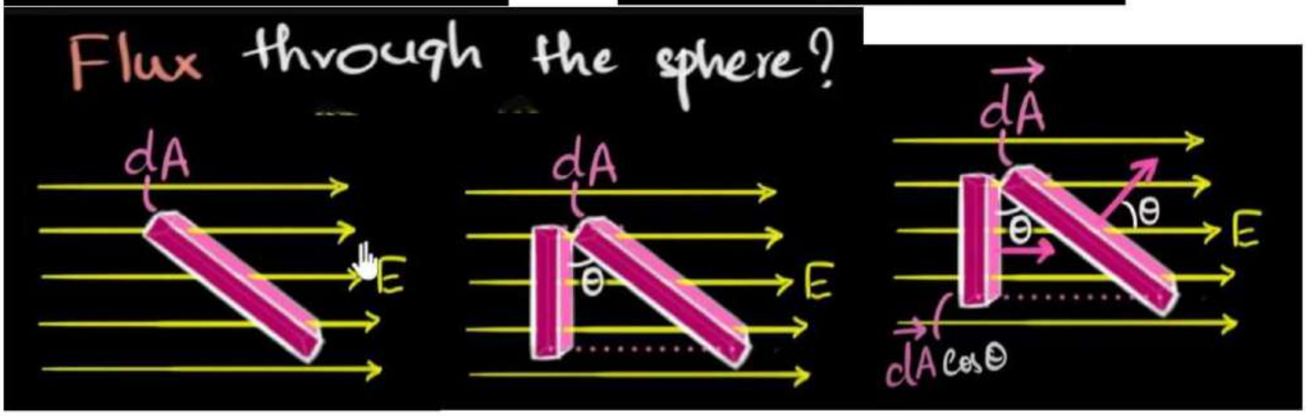
$$\vec{E} = \sum_i \vec{E}_i \rightarrow \phi = \sum_i \phi_i$$

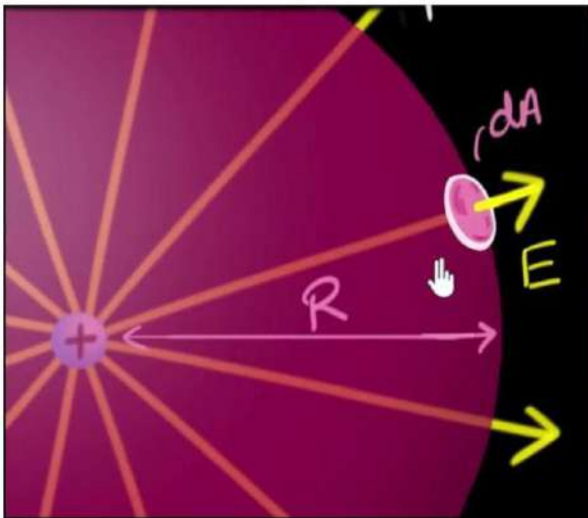
For n charges inside the closed surface:

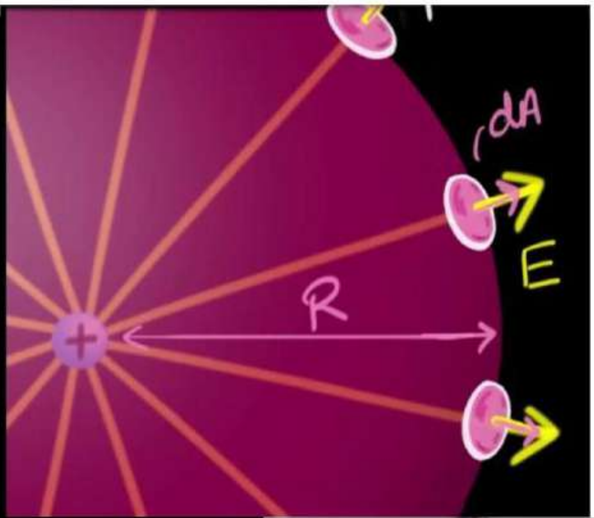
$$\boxed{\phi = \frac{1}{\varepsilon_0}\sum_i^n q_i\,(int)}$$






Flux through the sphere?
dA
E
dA
θ
E
dA
θ
θ
E
dA cosθ





$$\phi = \int \vec{E}\cdot d\vec{A} = \int E\,dA\cos\theta$$

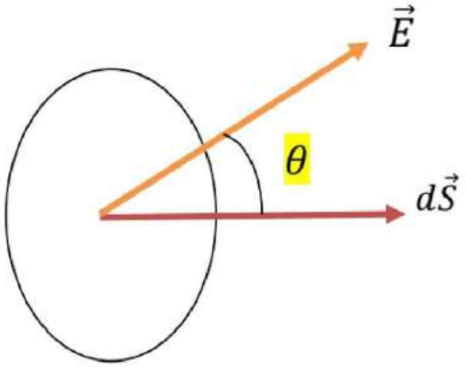
$$\phi = \int \vec{E}\cdot d\vec{A} = \int \underset{\text{const}}{E}\,dA\cos\underset{0}{\theta}$$

$$\phi = E\int dA$$

$$\phi = \frac{q}{4\pi\varepsilon_0 R^2}\times 4\pi R^2$$

$$\phi = \frac{q}{\cancel{4\pi}\varepsilon_0 \cancel{R^2}}\times \cancel{4\pi}\cancel{R^2}$$

$$\boxed{\phi = q/\varepsilon_0}$$

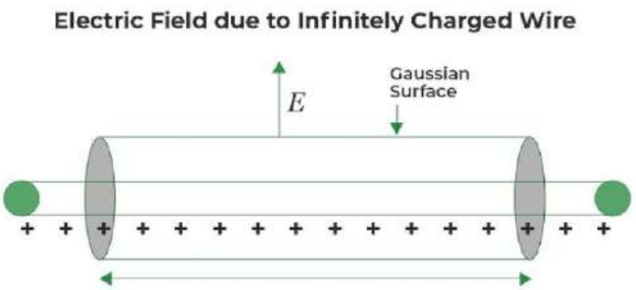


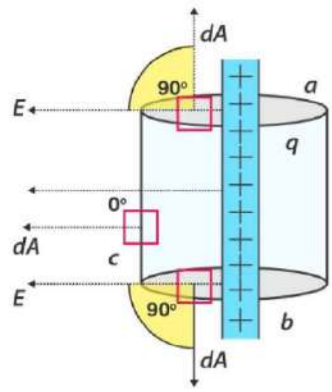
$$\theta = 0 \rightarrow \cos\theta = 1$$

$$\boxed{\oiint_{S_G} \vec{E}.\vec{n}.dS_G = \frac{1}{\varepsilon_0}\sum_i q_i\,(int)}$$

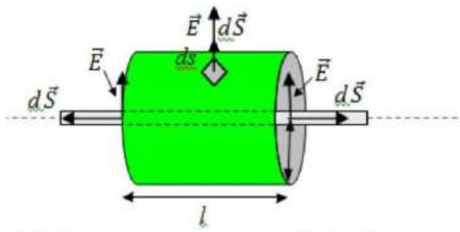
Choice of the Gaussian surface:

1. **The Gaussian surface must be closed.**
2. **The surface must pass through the point at which we want to calculate the field.**
3. **At every point on this surface, the electric field vector must be parallel to the surface normal.**





$$\Phi = \oint_S \vec{E} \cdot \overrightarrow{dS} = \iint_S \vec{E} \cdot \overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0}$$



$$\Phi = \sum \Phi_i \qquad \Phi = \oint_S \vec{E} \cdot \overrightarrow{dS} = \oint_{S_1} \vec{E} \cdot \overrightarrow{dS_1} + \oint_{S_2} \vec{E} \cdot \overrightarrow{dS_2} + \oint_{S_L} \vec{E} \cdot \overrightarrow{dS}_{\mathrm{L}}$$

$$\vec{E} \perp \overrightarrow{dS} \Rightarrow \Phi_1 = \Phi_2 = \iint_{S_1} \vec{E} \cdot \overrightarrow{dS_1} = \iint_{S_2} \vec{E} \cdot \overrightarrow{dS_2} = \iint_{S_1} E \cdot dS_1 \cdot cos\frac{\pi}{2} = \iint_{S_2} E \cdot dS_2 \cdot cos\frac{\pi}{2} = 0$$

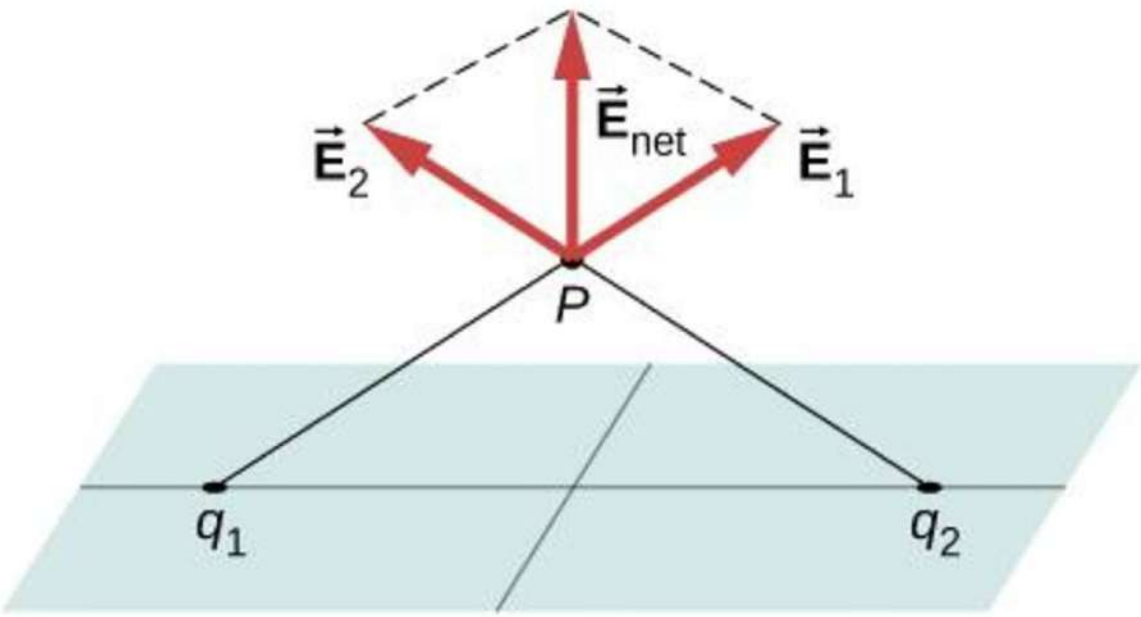
$$:(S_L) \Rightarrow \vec{E} // \overrightarrow{dS} \Rightarrow \quad \Phi_L = \iint_{S_L} \vec{E} \cdot \overrightarrow{dS_L} = \iint_{S_L} E \cdot dS_L \cdot cos\, 0 = E \cdot S_L$$

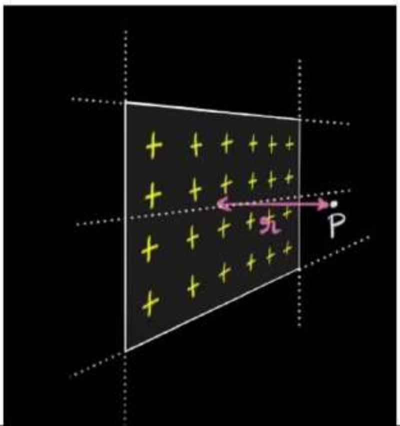
$$\boxed{\Phi = E \cdot S_L = \frac{Q_{int}}{\varepsilon_0}}$$

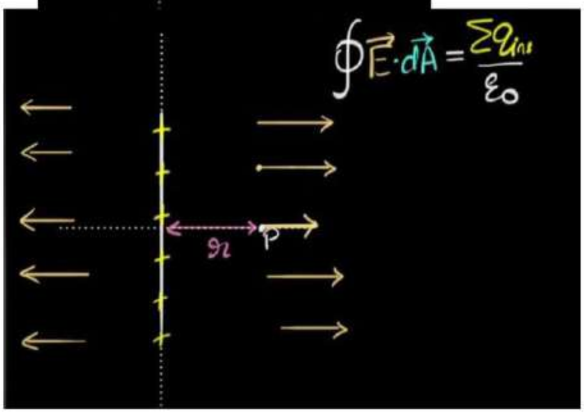
$$dq = \lambda . \mathrm{dl} = dQ_i \Rightarrow Q_{int} = Q_i = \boldsymbol{\lambda}.\mathrm{l}$$

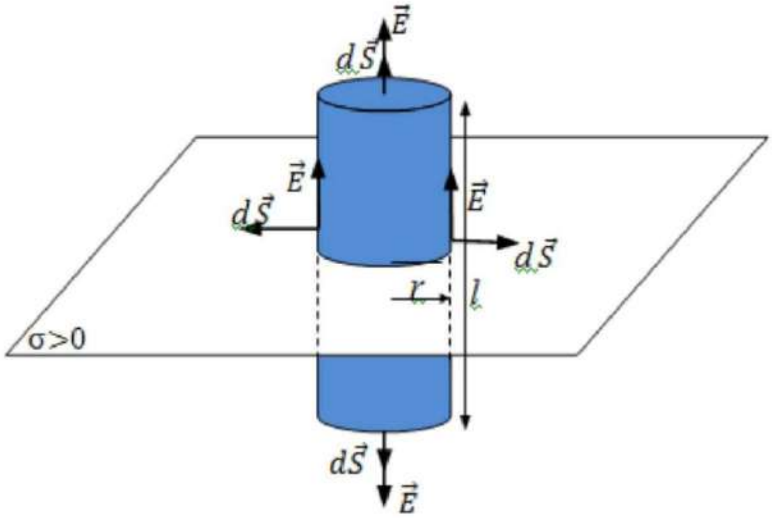
$$S_L = 2\pi Rl$$

$$E.2\pi Rl = \frac{\lambda.\mathrm{l}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\lambda}{2\pi R\varepsilon_0}}$$


$\vec{E}_{net}$
$\vec{E}_2$
$\vec{E}_1$
P
q_1
q_2


P


$\oint \vec{E}\cdot d\vec{A} = \frac{\Sigma q_{in}}{\varepsilon_0}$
P

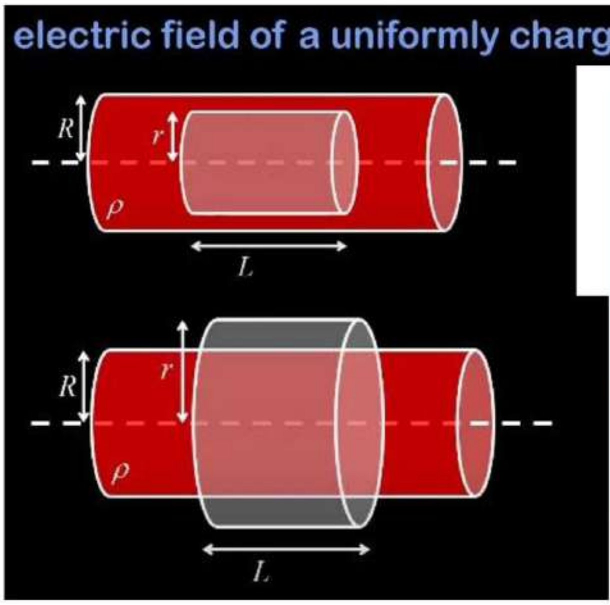

$\vec{E}$
$d\vec{S}$
$\vec{E}$
$d\vec{S}$
$\vec{E}$
$d\vec{S}$
r
l
σ>0
$d\vec{S}$
$\vec{E}$

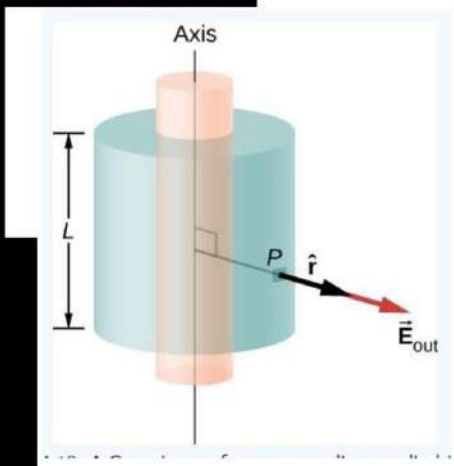
$$\Phi = \iint_{S_1} \vec{E} \cdot \overrightarrow{dS_1} + \iint_{S_2} \vec{E} \cdot \overrightarrow{dS_2} + \iint_{S_L} \vec{E} \cdot \overrightarrow{dS_L} = \frac{Q_{int}}{\varepsilon_0}$$

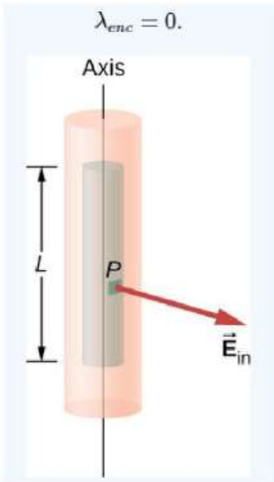
$S_1, S_2 = S \qquad \Phi = E.S_1 + E.S_2 + 0 = 2E.S \qquad (\vec{E}//\overrightarrow{dS_1}\ ,\ \vec{E}//\overrightarrow{dS_1}$ et $\vec{E} \perp \overrightarrow{dS_L})$

$Q_{int} = Q_i = \sigma.\mathrm{S} \qquad \Rightarrow \Phi = 2E.S = \dfrac{Q_{int}}{\varepsilon_0} = \dfrac{\sigma.S}{\varepsilon_0} \qquad \boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

$dV = -\vec{E}.\overrightarrow{dl} \qquad \Rightarrow \boxed{V = -\dfrac{\sigma}{2\varepsilon_0}.Z}$







$$\vec{E}_M = E_r(r).\vec{u}_r$$

$$\underbrace{\oiint_{S_G} \vec{E}.\vec{n}.dS_G}_{T_1} = \underbrace{\frac{1}{\varepsilon_0}\sum_i q_i\,(int)}_{T_2}$$

Surface charged cylinder

$\sigma = \frac{Q_{int}}{S_L} \rightarrow Q_{int} = \sigma \,.\, S_L \qquad = \sigma\, 2\, \pi\, \mathrm{R}\, \mathrm{h}$

$r > R$

$\varphi = \mathrm{E}\,(\mathrm{r})\,.\,2\,\pi\,\mathrm{r}\,\mathrm{h} \qquad \mathrm{E}\,(\mathrm{r}) = \frac{\sigma R}{\varepsilon_0 r}$

$Q_{int} = 0 \quad \mathrm{E}\,(\mathrm{r}) = 0$

$r < R$

$\Phi = \oint_S \vec{E} \cdot \overrightarrow{dS} = \iint_S \vec{E} \cdot \overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0}$

Volume charged cylinder

$\rho = \frac{Q_{int}}{V} \rightarrow Q_{int} = \rho \,.\, \mathrm{V} = \rho\, \pi\, R^2\, \mathrm{h}$

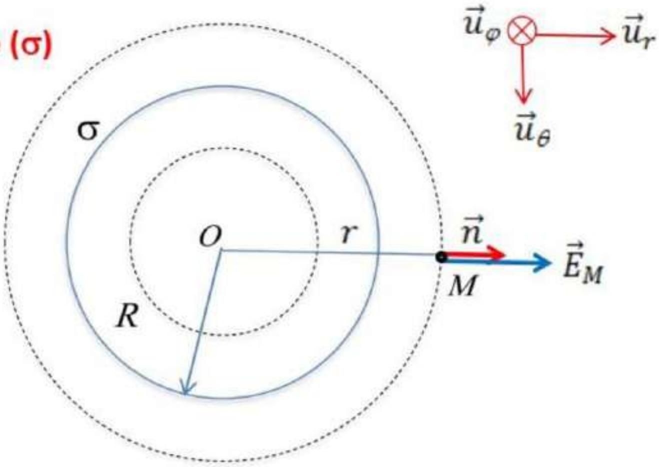
$r > R$

$\varphi = \mathrm{E}\,(\mathrm{r})\,.\,2\,\pi\,\mathrm{r}\,\mathrm{h} \qquad \mathrm{E}\,(\mathrm{r}) = \frac{\rho R^2}{2\,\varepsilon_0 r}$

$\rho = \frac{Q_{int}}{V} \rightarrow Q_{int} = \rho \,.\, \mathrm{V} = \rho\, \pi\, r^2\, \mathrm{h} \qquad \varphi = \mathrm{E}\,(\mathrm{r})\,.\,2\,\pi\,\mathrm{r}\,\mathrm{h} \quad \mathrm{E}\,(\mathrm{r}) = \frac{\rho r}{2\,\varepsilon_0}$

$r < R$

sphère (σ)



$$\vec{E}_M = E_r(r).\vec{u}_r + E_\theta(r).\vec{u}_\theta + E_\varphi(r).\vec{u}_\varphi$$

$$\vec{E}_M = E_r(r).\vec{u}_r$$

$$= E_r(r).S_G = E_r(r).4\pi r^2$$

$$\frac{1}{\varepsilon_0}\sum_i q_i\,(int) = \frac{\sigma.4\pi R^2}{\varepsilon_0}$$

$$E_r(r).4\pi r^2 = \frac{\sigma.4\pi R^2}{\varepsilon_0}$$

$$\boxed{E_r(r) = \frac{\sigma R^2}{\varepsilon_0 r^2} \rightarrow \vec{E}_M = \frac{\sigma R^2}{\varepsilon_0 r^2}\vec{u}_r}$$

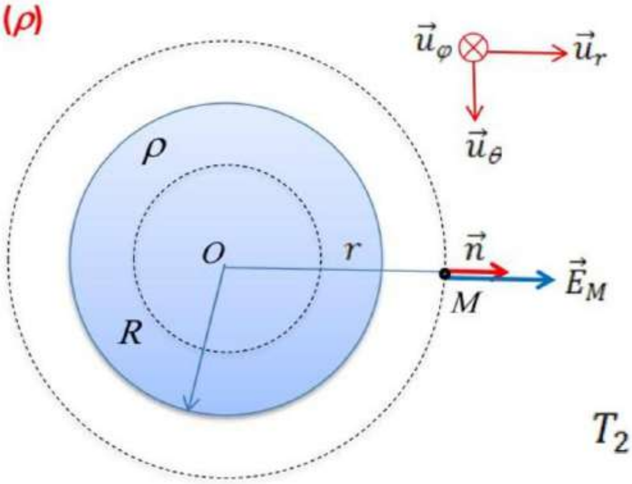
$r > R$

$$T_2 = \frac{1}{\varepsilon_0}\sum_i q_i\,(int) = 0 \qquad E_r(r).4\pi r^2 = 0$$

$$\boxed{E_r(r) = 0\ ,\ \vec{E}_M = \vec{0}}$$

$r < R$

La boule (ρ)



$$\vec{E}_M = E_r(r).\vec{u}_r + E_\theta(r).\vec{u}_\theta + E_\varphi(r).\vec{u}_\varphi$$

$$\vec{E}_M = E_r(r).\vec{u}_r$$

$$E_r(r).S_G = E_r(r).4\pi r^2$$

$$T_2 = \frac{1}{\varepsilon_0}\sum_i q_i\,(int) = \frac{\rho.4\pi R^3}{3\varepsilon_0}$$

$$E_r(r).4\pi r^2 = \frac{\rho.4\pi R^3}{3\varepsilon_0} \qquad \boxed{E_r(r) = \frac{\rho R^3}{3\varepsilon_0 r^2} \rightarrow \vec{E}_M = \frac{\rho R^3}{3\varepsilon_0 r^2}\vec{u}_r}$$

$$T_2 = \frac{1}{\varepsilon_0}\sum_i q_i\,(int) = \frac{\rho.4\pi r^3}{3\varepsilon_0} \qquad E_r(r).4\pi r^2 = \frac{\rho.4\pi r^3}{3\varepsilon_0} \qquad \boxed{E_r(r) = \frac{\rho r}{3\varepsilon_0} \rightarrow \vec{E}_M = \frac{\rho r}{3\varepsilon_0}\vec{u}_r}$$