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Sample Correction for Series N°02.
Algebra 1

Exercise 01 :

1. **Well-defined set** : $A = \{x \in \mathbb{R}, x^2 \geq 0\} = \mathbb{R}$

2. **Well-defined set** : $B = \{x \in \mathbb{R}, x^2 \leq 1\} = [-1, 1]$

3. **Well-defined set** : $C = \{x \in \mathbb{R}, x^2 = 1\} = \{-1, 1\}$

4. **Empty set** : $D = \{x \in \mathbb{R}, x^2 + 1 = 0\} = \emptyset$
   No real number satisfies $x^2 = -1$.
5. **Not well-defined** : $E = \{$beautiful flowers in the garden$\}$
   "Beautiful" is subjective and ambiguous.
6. **Well-defined set** : $F = \{x \in \mathbb{Z}, -2 \leq x < 4\} = \{-2, -1, 0, 1, 2, 3\}$
   Finite set of integers in the specified range.
7. **Empty set** : $G = \{x \in \mathbb{R}, x < x\} = \emptyset$
   No real number is less than itself.
8. **Not well-defined** : $H = \{$the best students in the class$\}$
   "Best" is subjective and lacks clear criteria.
9. **Well-defined set** : $I = \{x \in \mathbb{R}, x^3 - x = 0\} = \{-1, 0, 1\}$
   Solutions to $x(x-1)(x+1) = 0$.
10. **Well-defined set** : $J = \{x \in \mathbb{C}, x^2 = -4\} = \{-2i, 2i\}$

11. **Empty set** : $K = \{p, p$ is a polygon with 2 sides$\} = \emptyset$
    By definition, a polygon must have at least 3 sides.
12. **Not well-defined** : L is the set of large numbers.
    "Large" is relative and ambiguous.
13. **Well-defined set** : $M = \{n \in \mathbb{N}, n$ is even and prime$\} = \{2\}$
    2 is the only even prime number.
14. **Well-defined set** : $N = \{(x, y) \in \mathbb{R}^2, x^2 + y^2 = 1\}$
    The unit circle - infinite set of points.
15. **Well-defined set** : $O = \{y \in \mathbb{R}, |y| \leq 3\} = [-3, 3]$

16. **Empty set** : $P = \{x \in \mathbb{R}, \sin x = 2\} = \emptyset$
Sine function range is $[-1, 1]$.

Exercise 02 :

1. Let $E = \{-2, -1, 0, 1, 2, 3, a, e, f\}$.
   - $0 = \emptyset$ is false, we suppose that $0 = \emptyset \Rightarrow 0 \in \emptyset$ (this is a contradiction), then $0 \neq \emptyset$
   - $1 \subseteq E$ is false (1 is an element, isn't a set), we write
   $$1 \in E$$
   - $\emptyset \in E$ is false ($\emptyset$ is not an element of E)
   - $\emptyset \subseteq E$ is true (because $\emptyset$ is a subset of any set)
   - $\emptyset \cup E = E$ is true (because $\emptyset \cup E = \{x, x \in E \text{ or } x \in \emptyset\}$)
   $$\text{Where } \emptyset \subseteq E \Rightarrow \emptyset \cup E = E$$
   - $\emptyset \cap E = \emptyset$ is true (because $\emptyset \cap E = \{x, x \in E \wedge x \in \emptyset\}$)
   $\emptyset \cap E = \{x, x \in E \wedge x \in \emptyset \subseteq E\}$
   But $\emptyset$ has no element.
   $\emptyset \cap E = \{x, x \in E \wedge x \notin \emptyset\} = \emptyset$
2. Let $B = \{-1, 0, e, f\}$, $C = \{x \in \mathbb{R} \mid -2 \leq x < 3\}$, $D = \{a, c, e\}$:

$$B \cap D = \{x, x \in B \wedge x \in D\} = \{e\}$$
$$B \cup D = \{x, x \in B \vee x \in D\} = \{-1, 0, a, c, e, f\}$$
$$C_E(B) = \{x, x \in E \wedge x \notin B\} = \{-2, 1, 2, 3, a\}$$
$$C_E(D) = \{x, x \in E \wedge x \notin D\} = \{-2, -1, 0, 1, 2, 3, f\}$$
$$E - B = \{x, x \in E \wedge x \notin B\} = \{-2, 1, 2, 3, a\} = C_E(B)$$
$$B \triangle D = \{x, (x \in B \wedge x \notin D) \vee (x \in D \wedge x \notin B)\}$$
$= \{-1, 0, f\} \cup \{a, c\} = \{-1, 0, a, c, f\}$.

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   $$B \cap C = \{x, x \in B \wedge x \in C\} = \{-1, 0\}$$
   $$B \cup C = \{x, x \in B \vee x \in C\} = \{x \in \mathbb{R} \mid -2 \leq x < 3\} \cup \{e, f\}$$
   $$C_E(C) = \{x, x \in E \wedge x \notin C\} = \{3, a, e, f\}$$
   $$B \triangle C = \{x, (x \in B \wedge x \notin C) \vee (x \in C \wedge x \notin B)\}$$
   $$= \{e, f\} \cup \{x \in \mathbb{R} \mid -2 \leq x < -1 \vee 0 < x < 3\}$$
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   $$B \times D = \{(x, y), x \in B \wedge y \in D\} = \{(-1, a), (-1, c), (-1, e), (0, a), (0, c), (0, e), (e, a), (e, c), (e, e), (f, a), (f, c), (f, e)\}$$
   $$B \times \emptyset = \{(x, y), x \in B \wedge y \in \emptyset\} = \emptyset$$
   $$\mathcal{P}(B) = \{X, X \subseteq B\} = \{\emptyset, \{-1\}, \{0\}, \{e\}, \{f\}, \{-1, 0\}, \{-1, e\}, \{-1, f\}, \{0, e\}, \{0, f\}, \{e, f\}, \{-1, 0, e\}, \{-1, 0, f\}, \{-1, e, f\}, \{0, e, f\}, \{-1, 0, e, f\}\}$$
   $$\mathcal{P}(D) = \{X, X \subseteq D\} = \{\emptyset, \{a\}, \{c\}, \{e\}, \{a, c\}, \{a, e\}, \{c, e\}, \{a, c, e\}\}$$

Remark : The number of elements of the power set $\mathcal{P}(E)$ of a set E is given by the formula 2^n where n is the number of the elements of E.

- $C_D(B) = \{x \in D \mid x \notin B\} = \{a, c\}$.

Exercise 03 :

1. Let $A \cup B = A \cap B$ prove that $A = B$.
   If $x \in A \implies x \in A \cup B = A \cap B \subseteq B \Rightarrow x \in B$
   Then $A \subseteq B$
   By the same reasoning, we get $B \subseteq A$ and hence $A = B$.

2. "$\Rightarrow$" let $A \cap B = \emptyset$, if $x \in A \implies x \notin B$ i.e. $x \in C_E^B$ That's $A \subseteq C_E^B$
   "$\Leftarrow$" let $A \subset C_E^B \implies A \cap B = \emptyset$?
   Let $x \in A \cap B \implies x \in A \wedge x \in B \Rightarrow x \in C_E^B \wedge x \in B \Rightarrow x \in \emptyset$
   Then, $\phi \cap A \cap B \subseteq \emptyset$
   So $A \cap B = \emptyset$

3. "$\Rightarrow$" Let $A \subset B$
   if $x \in C_E^B \Rightarrow x \notin B$
   $\Rightarrow x \notin A$
   $\Rightarrow x \in C_E^A$. Thus $C_E^B \cap C_E^A$
   "$\Leftarrow$" if $x \in A \Leftrightarrow x \notin C_E^A$
   $\Rightarrow x \notin C_E^B$
   $\Rightarrow x \in B$

4. $C_E(C_E(A)) = A$.
   "$\subseteq$". Let $x \in C_E(C_E(A)) \Rightarrow x \notin C_E(A)$
   $$\Rightarrow x \in A$$
   $$\Rightarrow C_E(C_E(A)) \subseteq A$$
   "$\supseteq$". Let $x \in A \Rightarrow x \notin C_E(A)$
   $$\Rightarrow x \in C_E(C_E(A))$$
   $\Rightarrow A \subseteq C_E(C_E(A))$, and hence the equality

5. $x \in A \triangle B$
   $$\begin{aligned}
   &\Leftrightarrow (x \in A \wedge x \notin B) \cup (x \in B \wedge x \notin A)\\
   &\Leftrightarrow (x \in A \wedge x \in C_E^B) \cup (x \in B \wedge x \in C_E^A)\\
   &\Leftrightarrow (x \notin C_E^A \wedge x \in C_E^B) \cup (x \notin C_E^B \wedge x \in C_E^A)\\
   i.e. \quad &(x \in C_E^A \wedge x \notin C_E^B) \cup (x \in C_E^B \wedge x \notin C_E^A)\\
   &\Leftrightarrow x \in C_E^A \triangle C_E^B
   \end{aligned}$$

6. $(A \times C) \cup (B \times C) = (A \cup B) \times C$.

   $(x, y) \in (A \times C) \cup (B \times C) \iff (x, y) \in A \times C \vee (x, y) \in B \times C$

   $$\begin{aligned} &\Leftrightarrow (x \in A \wedge y \in C) \quad \vee \quad (x \in B \wedge y \in C) \\ &\Leftrightarrow (x \in A \vee x \in B) \wedge y \in C \\ &\Leftrightarrow x \in (A \cup B) \wedge y \in C \\ &\Leftrightarrow (x, y) \in (A \cup B) \times C \end{aligned}$$

7. Let $A \subset B$.

   If $H \in \mathcal{P}(A) \Rightarrow H \subset A \subset B$

   $$\Rightarrow (H \subset B) \Rightarrow H \in \mathcal{P}(B)$$

   Thus, $\mathcal{P}(A) \subset \mathcal{P}(B)$