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First year in Computer Science

Department of Computer Science
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**Exam of Algebra 1** (Normal Session) **Duration : 2h**

**Exercise 1 (5 pts)**

1. Draw the truth table for the following proposition : $(\overline{p} \vee q) \wedge \overline{r} \Leftrightarrow \overline{(p \wedge \overline{q}) \vee r}$.
2. Express the following proposition using (quantifiers and logical connectives), and give its negation :
   - $p$ : for every integer number $x$, if $(x-1)^2$ is even then $x$ is odd.
3. Recall the contrapositive law of logic.
4. Using the contrapositive law, prove that $p$ is true.

**Exercise 2 (4 pts).**

- We define the binary relation $\mathcal{R}$ on $\mathbb{R}^*$ as follows : $x\mathcal{R}y \iff x^2 - \dfrac{1}{x^2} = y^2 - \dfrac{1}{y^2}$
  1. Show that $\mathcal{R}$ is an equivalence relation.
  2. Determine the equivalence class of 1 i.e, ([1]), and deduce the equivalence class of $-1$.
  3. Find the equivalence class for every $x \in \mathbb{R}^*$.

**Exercise 3 (8 pts)**

**I.** Consider the internal operation $*$ on $\mathbb{R} \setminus \{2\}$ defined by :

$$\forall x, y \in \mathbb{R} \setminus \{2\}, \quad x * y = \frac{2x + 2y - xy}{2}$$

- Show that $(\mathbb{R} \setminus \{2\}, *)$ is a commutative group.

**II.** Let $f$ be a function defined by

$$f : (\mathbb{R} \setminus \{2\}, *) \to (\mathbb{R}^*, \times), \quad x \mapsto f(x) = \tfrac{2-x}{2},$$

with $\times$ is the usual multiplication.

1. Determine $\mathrm{Ker}(f)$ and $\mathrm{Im}(f)$.
2. Prove that $f$ is a homomorphism.
3. Is $f$ an isomorphism ?
4. Compute $f([-2, 0])$, $f^{-1}(1)$.

**Exercise 4 (3 pts).** Let $\mathcal{R}$ be an equivalence relation on a set $E$. Prove the following :

1. $\forall a \in E,\ a \in [a]$.
2. $\forall a, b \in E,\ a\mathcal{R}b \Leftrightarrow [a] = [b]$.