Chapter 4: Magnetostatics

1. Introduction

Magneto-statics is a branch of electromagnetism that studies time-invariant magnetic fields—fields produced by constant electric currents (that do not change with time $\Rightarrow \frac{dI}{dt} = 0$ *). It is considered a magnetic counterpart to electrostatics, which studies electric fields produced by stationary charges.*

A static electric charge produces an electric field, while a moving electric charge produces, in addition to the electric field, a magnetic field related to the direction and speed of the charge's movement.

2. Electromagnetic force (Lorentz's law)

This law states that every charge (q) moving at a velocity $\vec{v}$ *within an electromagnetic field will be subject to the influence of the magnetic field as well as the electric field. The expression for this force is written as follows:*

$$\vec{F} = \overrightarrow{F_{elec}} + \overrightarrow{F_{mag}} \quad where \begin{cases} \overrightarrow{F_{elec}} = q\,\vec{E} \\ \overrightarrow{F_{mag}} = q\,\vec{v} \times \vec{B} \end{cases}$$

3. Properties of magnetic force:

- *Magnetic force is nonexistent if the charge is stationary.*
- *Magnetic force is directed perpendicular to both the velocity and magnetic field.*
- *The magnetic force is zero if the charge's velocity vector is parallel to the magnetic field vector.*
- *The magnetic force reaches its maximum value when the charge's velocity vector is perpendicular to the magnetic field vector.*

- *The magnetic field does not change the velocity of the moving charge, but rather changes its direction.*
- ***Kinetic energy is conserved :*** *the change in kinetic energy of a charged particle moving within a magnetic field is equal to zero*

$$\Delta E_K(A \to B) = W(A \to B) = \int_A^B \overrightarrow{F_{mag}}\,.\overrightarrow{dr} = \int_A^B (q\,\vec{v} \times \vec{B}\,)\,.\overrightarrow{dr}$$

$$= \int_A^B (q\,\vec{v} \times \vec{B}\,)\,.\overrightarrow{dr}\frac{dt}{dt} = \int_A^B \left(q\,\vec{v} \times \vec{B}\,\right).\frac{\overrightarrow{dr}}{dt}dt$$

$$\int_A^B \left(q\,\vec{v} \times \vec{B}\,\right).\vec{v}\,dt \ = 0$$

$$\Delta E_K(A \to B) = 0 \Rightarrow E_K(B) = E_K(A)$$

4. The effect of a magnetic field on an electric current

Considering that the electric current flows through a group of charges.

*Each element length **dl** of wire passes through a current **I** and **N** electrons pass through it.*

$$\boldsymbol{N = n\,dV = n\,S\,dl}$$

*Where **S** is Cross-sectional area of wire, and **n** is the concentration of charges in the volume of the element dv. The expression for the magnetic force acting on the longitudinal element dl through which a current I passes is given by:*

$$\overrightarrow{F_{mag}} = q\,\vec{v} \times \vec{B}$$

$$d\overrightarrow{F_{mag}} = (-n\,e\,\vec{v}\,)S\,d\vec{l} \times \vec{B}$$

$$d\overrightarrow{F_{mag}} = I\,d\vec{l} \times \vec{B}$$

$$\overrightarrow{F_{mag}} = L\,\vec{I} \times \vec{B}$$

As we learned in the previous chapter

$$I = \vec{J}.\overrightarrow{dS}$$

So,

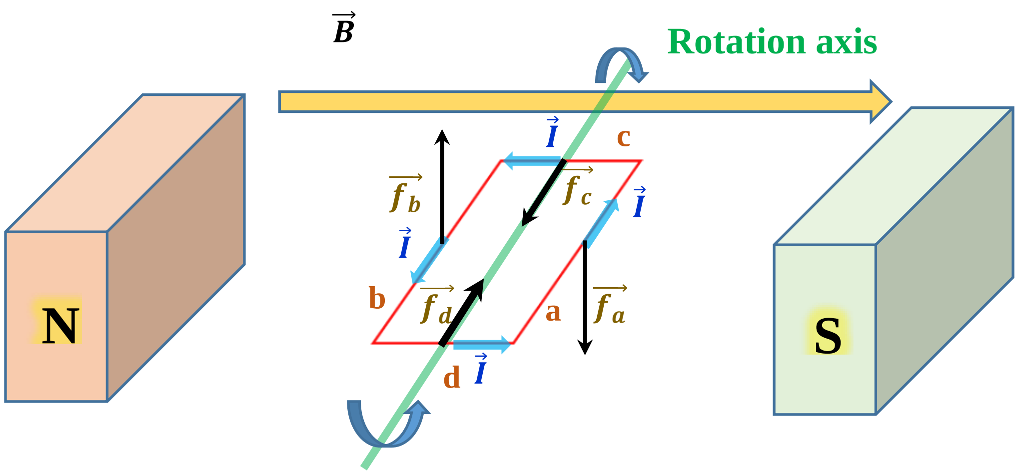
$$d\overrightarrow{F_{mag}} = (\vec{J}.\overrightarrow{dS})d\vec{l} \times \vec{B}$$

$$d\overrightarrow{F_{mag}} = \vec{J}.\overrightarrow{dS}\, d\vec{l} \times \vec{B}$$

$$d\overrightarrow{F_{mag}} = (\vec{J} \times \vec{B})\, dV$$

5. Torque on a Current Loop

The following figure represents a conducting wire structured as a rectangular loop with sides ABCD (the side's length of each A and B are labeled as L, and the sides C and D are labeled as K). A constant electric current flows through this loop. The loop is free to move and can rotate about an axis passing through the middle of sides D and C. The loop is placed under the influence of a constant magnetic field in the x-direction, parallel to sides C and D.

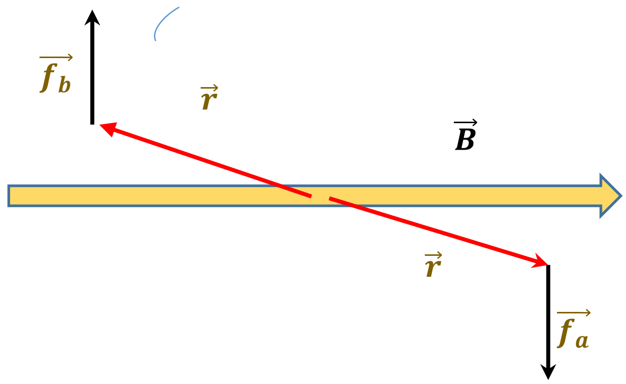


The magnetic field exerts a magnetic force on the electric current passing through each side.

By representing the forces, it is clear that the two forces $\vec{f_c}$ and $\vec{f_d}$ have zero torque of force.

The two magnetic forces applied to sides A and B form a double torque. The value of each force is given as follows:

$$\|\overrightarrow{f_b}\| = \|\overrightarrow{f_a}\| = L \times I \times B$$



$$\overrightarrow{\mathcal{M}}_{/0}(\overrightarrow{f_a}) = \vec{r} \wedge \overrightarrow{f_a} \;\; and \; \overrightarrow{\mathcal{M}}_{/0}(\overrightarrow{f_b}) = \vec{r} \wedge \overrightarrow{f_b}$$

$$\|\vec{r}\| = \frac{K}{2}$$

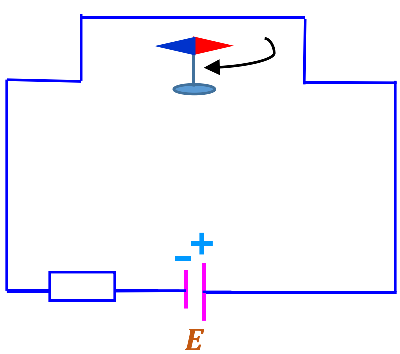
$$\overrightarrow{\mathcal{M}}_{/0} = \overrightarrow{\mathcal{M}}_{/0}(\overrightarrow{f_a}) + \overrightarrow{\mathcal{M}}_{/0}(\overrightarrow{f_b})$$

$$\|\overrightarrow{\mathcal{M}}_{/0}\| = \frac{K}{2} L \times I \times B \sin\theta + \frac{K}{2} L \times I \times B \sin\theta = K\, L \times I \times B \sin\theta$$

This idea is the working principle of electromagnetic motors.

6. Oersted's experiment:

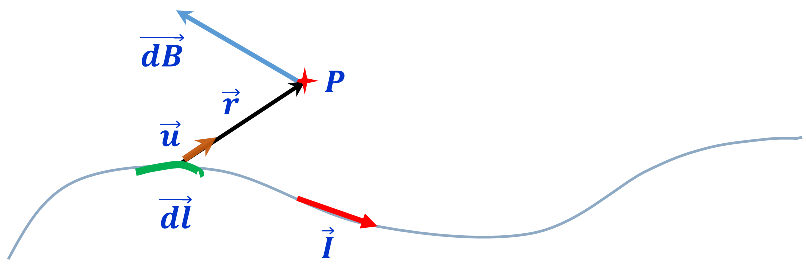
In 1820, Oersted observed a magnetic needle deflect when placed next to a wire carrying an electric current. It then returned to its normal position when the electric current was cut off. This experiment was the first step in generating a magnetic field through induction.



It was also shown through the same experiment that the direction of deflection of the magnetized needle is related to the direction of the electric current.

7. Magnetic field created by a current (The Biot–Savart Law)

This law states that a magnetic field is generated at a point ***P*** *in the space surrounding a conducting wire (Any electric charge moving at a speed of* $\boldsymbol{v}$ *) as a result of an electric current passing through it. This phenomenon is called "magnetic induction".*



The expression for the elemental magnetic field formed by the passage of an electric current ***I*** *through an element of length* ***dl*** *at any point* ***P*** *in space is as follows:*

$$d\vec{B} = \frac{\mu_0\, I}{4\,\pi\, r^2}\,.\overrightarrow{dl}\times\vec{u}$$

For an electric charge moving at a velocity of $\vec{v}$

$$\vec{B} = \frac{\mu_0\, q}{4\,\pi\, r^2}\,.\vec{v}\times\vec{u} = \frac{\mu_0\, q}{4\,\pi\, r^3}\,.\vec{v}\times\vec{r}$$

μ_0 *denotes a constant called the permeability of free space', its value equal to* $\boldsymbol{4\,\pi\, 10^{-7}\, T.m/A}$