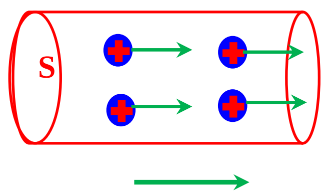
Chapter 3: Electric Circuits (Electrokinetics

1) Electric current

Electric current is defined as the instantaneously collective and orderly movement of electric charges within a conductor, occurring when the conductor is subjected to an electric field created by a potential difference between its ends. This flow of charges constitutes an electric current, whose intensity is measured as the amount of charge passing through a given cross-sectional area per unit time. The direction of the conventional current is taken to be the direction positive charges would move, even though in most conductors the actual charge carriers are electrons moving in the opposite direction.



Electric current intensity (I) is defined as the amount of electric charge (Q) that passes through the surface of a conductor during a unit of time (t). Its unit in the ***SI*** *unit is* ***ampere (A)*** *and its expression is given as :*

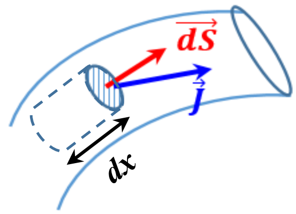
$$I = \frac{Q}{t} = \frac{dQ}{dt}$$

2) Electric Current Density

Electric Current Density *is a vector quantity that expresses the magnitude and direction of the electric current passing through a unit area perpendicular to the direction of the current at a given point within the conductor. It shows how the current is distributed within the conductor.*

The relationship between electric current and current density is written as follows:

$$i(t) = \int \vec{J}\,.\,\overrightarrow{dS}$$



The current density can be written in terms of the average velocity $\boldsymbol{v_a}$ *of the electric charges. Suppose that n charges are inside a cylinder of elemental volume (length* **dx** *and area* **dS***) and an electric current di flows inside it, the volume density of the charge inside it* $\boldsymbol{\rho}$*. On this basis, the number* $\boldsymbol{n_c}$ *of charges inside the cylinder of elemental volume* **dV** *that will pass through the surface element* **dS***, covering a distance* **dx***, is expressed as follows:*

$$dV = dS \times dx \quad where \begin{cases} dx = v_a \, dt \\ dq = \rho \, dV \end{cases} \Rightarrow dq = \rho \, v_a \, dt \, dS$$

$$dq = \rho \, \vec{v}_a \, dt \, \overrightarrow{dS} = i \, dt$$

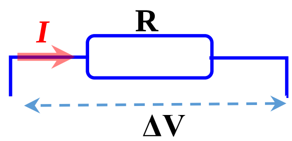
$$i = \vec{J}.\overrightarrow{dS} = \rho \, \vec{v}_a \, \overrightarrow{dS} \Rightarrow \vec{J} = \rho \, \overrightarrow{v_a}$$

If the number of free charges in a unit volume is n, then it is possible to write:

$$\rho = n \, q \Rightarrow \begin{cases} \vec{J} = n \, q \, \overrightarrow{v_a} \\ i = n \, q \, \overrightarrow{v_a} \vec{S} \end{cases}$$

3) Ohm's Law

Ohm's law states that the ratio of the potential difference between the two ends of a conductor to the electric current passing through it is a constant ratio. It is symbolized by **R** *and its unit is the* Ω.



*This ratio is called electrical resistance **R** and represents the electrical obstruction that a conductor shows to the movement of charges when an electric current passes through it.*

$$R = \frac{\Delta V}{I} = \frac{\int_A^B \vec{E}.\overrightarrow{dr}}{\int_S \vec{J}\,.\overrightarrow{dS}}$$

The simplified formula of Ohm's law is reached by finding the relationship that links the electric field applied to these moving charges and their speed. This is done by applying the basic principle of motion, where the charges during their movement are subject to two forces, the first of which is an electric driving force, given by expression (1), and the second is a frictional force, where its direction is opposite to the direction of the charge movement, and its intensity is proportional to the speed of the charges, given by expression (2).

$$\begin{cases} \overrightarrow{F_e} = q\,\vec{E} & (1) \\ \overrightarrow{F_f} = -k\ \vec{v} & (2) \end{cases}$$

$$\sum \overrightarrow{F_{ext}} = m\,\vec{a} = m\frac{d\vec{v}}{dt} \Rightarrow \quad \overrightarrow{F_e} + \overrightarrow{F_f} = q\,\vec{E} - k\ \vec{v} = m\frac{d\vec{v}}{dt}$$

$$q\,\vec{E} - k\ \vec{v} = m\frac{d\vec{v}}{dt}$$

If the current intensity is constant, the speed of charge movement will also reach a constant value, which is the entrained velocity, and therefore:

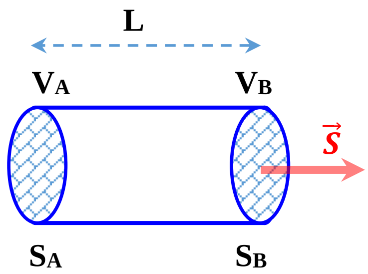
$$\frac{d\vec{v}}{dt} = 0\ \Rightarrow\ q\,\vec{E} = k\ \vec{v}\ \Rightarrow \vec{v} = \frac{q}{k}\,\vec{E} = \ \mu\ \vec{E}$$

*Multiplying both sides of this equation by **(n q)** we get*

$$n\,q\,\vec{v} = n\,q\,\mu\ \vec{E} = \vec{J} = \sigma\,\vec{E}\ where\ \sigma = \ n\,q\,\mu$$

σ is called conductivity of matter.

*In the case of a cylindrical conveyor with a cross-sectional area of **S** and a length of **L***



The electric field is uniform, so the electric potential difference between terminals A and B becomes:

$$\Delta V = V_B - V_A = \int_A^B \vec{E}.\overrightarrow{dr} = E\ L$$

The electric current intensity is written as follows:

$$I = \int_S \sigma\, \vec{E}\,.\overrightarrow{dS} = \sigma\ E\ S$$

$$R = \frac{V}{I} = \frac{E\ L}{\sigma\ E\ S} = \frac{L}{\sigma\ S} = \rho \frac{L}{S} \quad where\ \rho = \frac{1}{\sigma}$$

ρ *Represents the specific resistivity of the material.*

4) Electric circuits

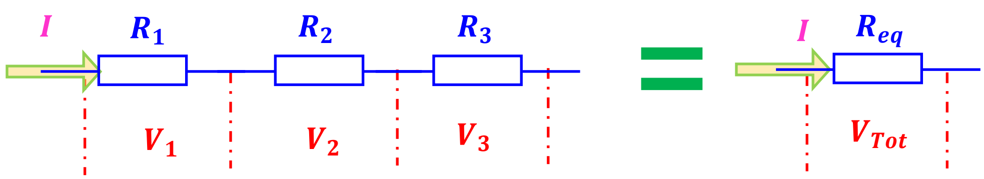
An electric circuit is an arrangement of interconnected electrical components that allows electric current to flow through them. These components may include sources of electromotive force (such as batteries or voltage supplies), resistors, inductors, capacitors, and semiconductor devices such as diodes and transistors.

5) Resistors combination

Resistors in electrical circuits can be arranged in series, in parallel, or in mixed combination.

6) 5.1) Series combination of Resistors

Resistors are installed in series combination when each pair of successive capacitors shares one connection point (one common terminal), and the same electric intensity passes through all resistors, while the potential difference across each one may vary.



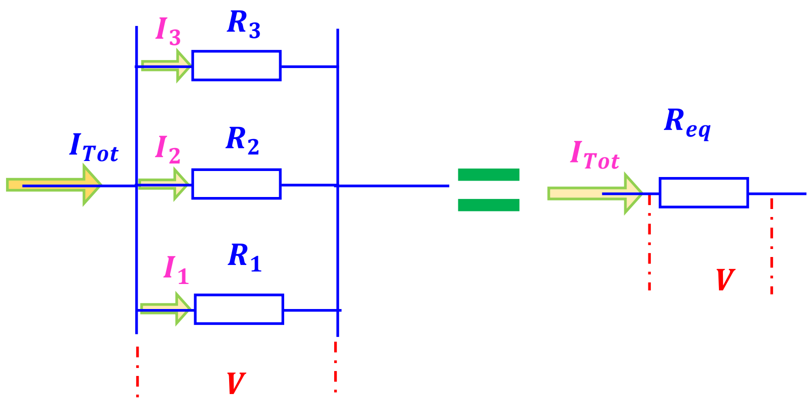
The electric current intensity passing through the equivalent resistor $\boldsymbol{R_{eq}}$ *is equal to the electric current intensity passing through the individual resistors* $\boldsymbol{R_i}$ *, meanwhile, the total potential difference* $\boldsymbol{V_{Tot}}$ *between the equivalent resistor ends is equal to the sum of the individual potential difference* $\boldsymbol{V_i}$ *between each resistor ends. The resistance value of the equivalent resistor can be calculated as follows:*

$$R_{eq} = \frac{\boldsymbol{V_{Tot}}}{I_{Tot}} \quad \begin{cases} \boldsymbol{I_{Tot}} = I_i \\ R_i = \dfrac{V_i}{I_i} \\ V_{Tot} = \sum_i V_i \end{cases}$$

$$R_{eq} = \frac{\boldsymbol{V_{Tot}}}{I_{Tot}} = \frac{\sum_i V_i}{I_i} = \sum_i \frac{V_i}{I_i} = \sum_i R_i$$

7) 5. 2) Parallel Combination of resistors

Resistors are connected in a parallel combination, all resistors are connected to the same two terminals, so the potential difference across each resistor is the same, while the total electric current is distributed among them according to their individual resistance value.



In the parallel capacitors combination method, the potential difference $\boldsymbol{V_{Tot}}$ *across the equivalent resistor* $\boldsymbol{R_{eq}}$ *is equal to the potential difference* $\boldsymbol{V_i}$ *across each individual resistor* $\boldsymbol{R_i}$*, while the total electric current* $\boldsymbol{I_{Tot}}$ *passes in the equivalent resistor is equal to the sum of the of the electric currents* $\boldsymbol{I_i}$ *passing through the individual resistors. The resistance of the equivalent resistor is calculated as follows:*

$$R_{eq} = \frac{\boldsymbol{V_{Tot}}}{I_{Tot}} \quad \begin{cases} \boldsymbol{I_{Tot}} = \sum_i I_i \\ R_i = \dfrac{V_i}{I_i} \\ V_{Tot} = V_i \end{cases}$$

$$R_{eq} = \frac{\boldsymbol{V_{Tot}}}{I_{Tot}} = \frac{V_i}{\sum_i I_i} \Rightarrow \frac{1}{R_{eq}} = \frac{\sum_i I_i}{V_i} = \sum_i \frac{I_i}{V_i} = \sum_i \frac{1}{R_i}$$

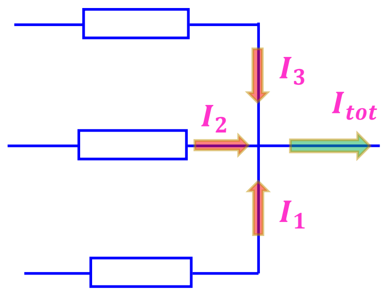
8) *Kirchhoff's laws*

To facilitate the study of electrical circuits and to know the current intensity passing through each branch of the electrical circuit, or the electrical voltage between the two ends of each element, we rely on Kirchhoff's laws.

9) *6.1) Junction rule:*

At any junction in a circuit, the sum of the ingoing currents must equal the sum of the outgoing currents. That is:

$$\sum I_{in} = \sum I_{out}$$

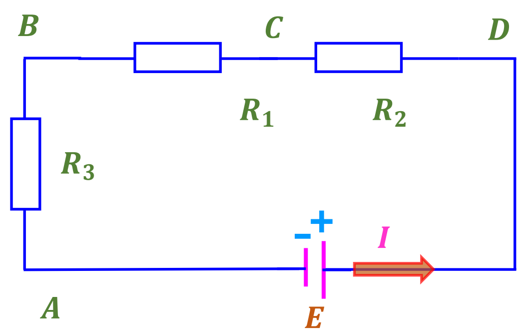


$$I_{tot} = I_1 + I_2 + I_3$$

10) 6.2) Loop rule:

For any closed loop in a circuit, the sum of the potential differences across all elements must be zero. That is:

$$\sum_{closed\ loop} V_I = 0$$



To study this circuit, we note the following:

- *The conventional direction of current is from the positive pole (**+**) to the negative pole (**-**) .*
- *When an electric current **I** passes through a resistor **R**, a potential difference **V** is generated between its two ends, the direction of which is opposite to the direction of the current.*
- *According to Ohm's law, this potential difference is equal to* $\boldsymbol{V = R \times I}$
- *The potential difference between the same point is zero* $\boldsymbol{V_{AA} = O}$
- *We mark the two ends of each electrical component with letters, as shown in the figure.*
- *We apply Kirchhoff's loop law to the circuit as follows:*

$$\sum_{closed\ loop} V_I = 0 \Rightarrow V_{AA} = 0$$

$$V_{AA} = V_{A\to D} + V_{D\to C} + V_{C\to B} + V_{B\to A}$$

We choose the following conventional direction $\boldsymbol{A \to D \to C \to B \to A}$ *as the positive directio:*

$$V_{A\to D} = E\ ;\ V_{D\to C} = -\boldsymbol{R_2} \times I\ ;\ V_{C\to B} = -\boldsymbol{R_1} \times I\ ;\ V_{B\to A} = -\boldsymbol{R_3} \times I$$

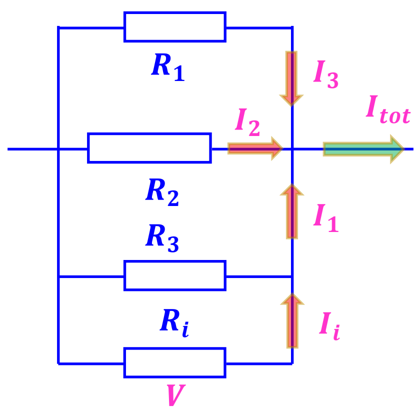
$$V_{AA} = 0 \Rightarrow E - \boldsymbol{R_2} \times I - \boldsymbol{R_1} \times I - \boldsymbol{R_3} \times I = 0$$

$$E = \boldsymbol{R_2} \times I + \boldsymbol{R_1} \times I + \boldsymbol{R_3} \times I$$

11) Current and Voltage Division Law

12) 7.1) Current Division Law

In a circuit that includes a group of resistors in parallel, the total electric current is divided according to the values of the resistors, where the resistor with the larger value has a smaller electric current passing through it.



$$I_{tot} = I_1 + I_2 + I_3 + I_i$$

$$I_i = \frac{V_i}{R_i} \quad and \quad V_i = V \quad \Rightarrow I_i = \frac{V}{R_i}$$

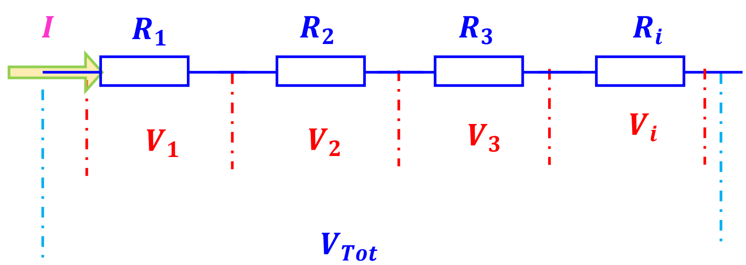
$$I_{tot} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_i} = V\ \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_i}\right) \Rightarrow V = \frac{I_{tot}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_i}\right)}$$

$$I_i = \frac{V}{R_i} = \frac{1}{R_i} \times \frac{I_{tot}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_i}\right)}$$

$$\boxed{I_i = \frac{1}{R_i} \times \frac{I_{tot}}{\sum_i \frac{1}{R_i}}}$$

13) 7.2) Voltage Division Law

In a circuit that includes a group of resistors in series, the total electrical voltage is divided according to the values of the resistors, where the resistor with the higher value has the greater voltage between its two ends.



$$V_i = R_i \times I_i \quad where\ I_i = I_{tot} = I = \frac{V_{tot}}{R_1 + R_1 + R_3 + R_i}$$

$$V_i = \frac{R_i}{R_1 + R_1 + R_3 + +R_i} \times V_{tot}$$

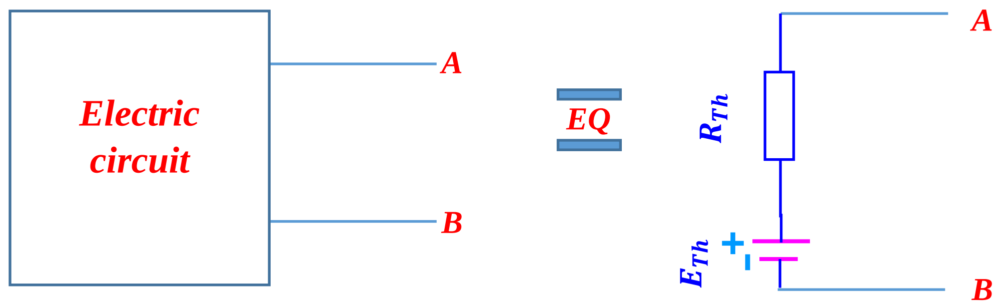
$$\boxed{V_i = \frac{R_i}{\sum_i R_i} \times V_{tot}}$$

14) Thevenin's and Norton's theorems

Thevenin and Norton methods are two of the most important methods for analyzing linear electrical circuits.

15) 8.1) Thévenin's theorem

Thévenin's theorem states that every linear circuit between two terminals A and B can be simplified into a simple circuit called a Thévenin circuit, which includes a Thévenin's generator E_{Th} *and a Thévenin's resistor* R_{Th} *connected in series.*

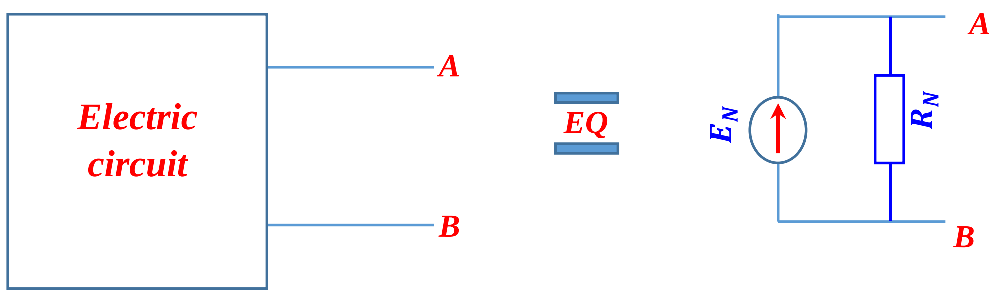


To convert any electrical network to its equivalent Thévenin circuit and to calculate the value of the Thévenin resistance and the electromotive force of the Thévenin generator, we perform the following steps:

- *Isolate the resistor to be measured.*
- *Short-circuit all voltage generators.*
- *Calculate the equivalent resistance.*
- *Calculate the potential difference between the two terminals of the branch.*

16) 8.2) Norton's theorems

The basis of Norton's theory is to convert an electrical circuit into a circuit consisting of a current generator (Norton generator) connected in parallel with a resistance (Norton resistor).



The steps followed in the Norton method are almost similar to the Thévenin method, where the Norton resistor is calculated in the same way as the Thévenin method, while the Norton current is calculated by shorting the target resistor in the calculation.