MOHAMED BOUDIAF UNIVERSITY – MSILA FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

LEVEL: FIRST YEAR **FIRST SEMESTER EXAM IN PHYSICS I MODULE** TIME: 2 HOURS

Exercise 01 : (5 points)

A) Using dimensional analysis, check whether the following equation is dimensionally correct :

$$\frac{1}{2}m\,v^2 = m\,g\,l\cos\theta \Rightarrow \begin{cases} \left[\frac{1}{2}m\,v^2\right] = \left[\frac{1}{2}\right][m][v^2] \\ [m\,g\,l\cos\theta] = [m][g][l][\cos\theta] \end{cases}$$

$$\left[\frac{1}{2}\right] = 1\,;\ [m] = M;\ [v^2] = L^2T^{-2};\ [g] = LT^{-2};\ [l] = L;\ [\cos\theta] = 1$$

$$\begin{cases} \left[\frac{1}{2}m\,v^2\right] = \left[\frac{1}{2}\right][m][v^2] = M\,L^2T^{-2} \\ [m\,g\,l\cos\theta] = [m][g][l][\cos\theta] = MLT^{-2}\,L = M\,L^2T^{-2} \end{cases}$$

$$\Rightarrow \left[\frac{1}{2}m\,v^2\right] = \left[\frac{1}{2}\right][m][v^2] \Rightarrow Eq\ correct$$ **(1.5p)**

B) The vectors $\vec{A}$ and $\vec{B}$ are given by : $\vec{A} = 2\vec{\imath} - \vec{\jmath} - \vec{k}$ $\quad\vec{B} = 2\vec{\imath} - \vec{k}$

$$\|\vec{A}\| = \sqrt{(2)^2 + (-1)^2 + (-1)^2} = \sqrt{6}$$ **(0.5p)**

$$\|\vec{B}\| = \sqrt{(2)^2 + (-1)^2} = \sqrt{5}$$ **(0.5 p)**

$$\vec{A}\cdot\vec{B} = (2\times 1) + ((-1)\times 0) + ((-1)\times(-1)) = 3$$ **(0.5 p)**

$$\vec{A}\wedge\vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & -1 & -1 \\ 2 & 0 & -1 \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix}\vec{\imath} - \begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix}\vec{\jmath} + \begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix}\vec{k} = (1-0)\vec{\imath} - (-2-(-2))\vec{\jmath} + (0-(-2))\vec{k} = \vec{\imath} + 2\vec{k}$$ **(1 p)**

1) Are $\vec{A}$ and $\vec{B}$ perpendicular or parallel? Justify your answer.

$$\begin{cases} \vec{A}\cdot\vec{B} \neq 0 \Rightarrow \theta \neq n\pi \\ \vec{A}\wedge\vec{B} \neq 0 \Rightarrow \theta \neq \frac{\pi}{2} + n\pi \end{cases} \Rightarrow$$ The two vectors $\vec{A}$ *and* $\vec{B}$ are neither parallel nor perpendicular **(1 p)**

Exercise 02 (4 points)

The coordinates of a moving particle $M(x, y)$ in a Cartesian coordinate system are given by:

$$x(t) = 2t \quad ; \quad y(t) = 5 + 5t^2$$

1. Find the equation of the trajectory (path).

$$x(t) = 2t \quad ; \quad y(t) = 5 + 5t^2 \Rightarrow t = \frac{x}{2} \Rightarrow y = 5 + \frac{5}{4}x^2$$ **(1 p)**

2. Write the expression of the position vector $\overrightarrow{OM}$.

$$\overrightarrow{OM} = x\,\vec{\imath} + y\,\vec{\jmath} = 2t\ \vec{\imath} + (5 + 5t^2)\,\vec{\jmath}$$ **(0.5 p)**

3. Calculate the average velocity between the instants $t_4 = 4s$ and $t_0 = 0s$.

$$\overrightarrow{V_{0,4}} = \frac{\overrightarrow{OM}\,(t_4 = 4s) - \overrightarrow{OM}\,(t_0 = 0s)}{t_4 - t_0} = \frac{(2\times 4\ \vec{\imath} + (5 + 5\times 4^2)\,\vec{\jmath}) - (2\times 0\ \vec{\imath} + (5 + 5\times 0^2)\,\vec{\jmath})}{4 - 0}$$

$$\overrightarrow{V_{0,4}} = \frac{8\ \vec{\imath} + 80\ \vec{\jmath}}{4 - 0} = 2\ \vec{\imath} + 20\,\vec{\jmath}$$ **(1 p)**

4. Write the expressions of the instantaneous velocity vector $\vec{v}$ and the instantaneous acceleration vector $\vec{a}$

$$\vec{v} = \frac{d\overrightarrow{OM}}{dt} = 2\ \vec{\imath} + (10t)\,\vec{\jmath}$$ **(0.75 p)**

$$\vec{a} = \frac{d\vec{v}}{dt} = 10\,\vec{\jmath}$$ **(0.75 p)**

Exercise 03 (4 points)

The coordinates of a moving particle $M(r, \theta)$ in a **polar coordinate system** with the unit vectors $\overrightarrow{u_r}$ and $\overrightarrow{u_\theta}$, are given by: $r = t^2$, $\quad \theta = \omega_0\,t$, where ω_0 represents a **constant angular velocity**.

1- In the polar coordinate system, write the expressions of the **position vector** $\overrightarrow{OM}$, the **velocity vector** $\vec{v}$, and the **acceleration vector** $\vec{a}$

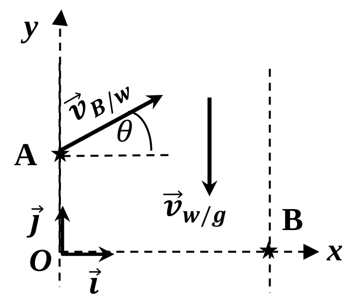
$$\overrightarrow{OM} = r\,\overrightarrow{u_r} = t^2\,\overrightarrow{u_r} \quad \textbf{(1 p)}$$

$$\vec{v}(t) = \frac{d\overrightarrow{OM}}{dt} = \frac{dr}{dt}\overrightarrow{u_r} + r\frac{d\theta}{dt}\vec{u}_\theta = \dot{r}\overrightarrow{u_r} + r\dot{\theta}\vec{u}_\theta = 2t\,\overrightarrow{u_r} + t^2\omega_0\vec{u}_\theta \quad \textbf{(1.5 p)}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{u}_\theta = (2 - t^2{\omega_0}^2)\vec{u}_r + (4t\omega_0)\vec{u}_\theta \quad \textbf{(1.5 p)}$$

Exercise 04 (7 points)

A boat crosses a river of width $L = 60\ m$ with a relative velocity of $\vec{v}_{B/w} = 2\ m/s$ to the water current. The boat departs from point A toward the opposite and moves at an angle $\theta = 30^0$ as shown in the figure. The velocity of the water current relative to the ground is equal to $\vec{v}_{w/g} = 1\,m/s$



1. Write the analytical expressions of: the **relative velocity vector** $\vec{v}_r$, the **entrained velocity vector** $\vec{v}_e$, then, deduce the expression of the **absolute velocity vector** $\vec{v}_a$

$$\overrightarrow{v_e} = \vec{v}_{w/g} = -1\,\vec{j} \quad ; \overrightarrow{v_r} = \vec{v}_{B/w} = 2\cos 30\ \vec{i} + 2\sin 30\ \vec{j} = \sqrt{3}\,\vec{i} + 1\,\vec{j}$$

$$\overrightarrow{v_a} = \overrightarrow{v_e} + \overrightarrow{v_r} = -1\,\vec{j} + \sqrt{3}\,\vec{i} + 1\,\vec{j} = \sqrt{3}\,\vec{i} \quad \textbf{(2 p)}$$

2. Determine the expression of the **position vector** $\overrightarrow{OM}$ of the boat with respect to the fixed reference frame (origin O), knowing that at $t = 0s$ the boat was at point $A(0\,;\,20\,m)$

$$\overrightarrow{v_a} = \sqrt{3}\,\vec{i} = \frac{d(\overrightarrow{OM})}{dt}$$

$$d(\overrightarrow{OM}) = (\sqrt{3}\,\vec{i})\ dt$$

$$\overrightarrow{OM}(t) = \int (\sqrt{3}\,\vec{i})\,dt = \sqrt{3}\ t\,\vec{i} + \vec{C}$$

at $t = 0s$ the boat was at point $A(0\,;\,20\,m)$ $\Rightarrow \overrightarrow{OM}(0) = 0\,\vec{i} + 20\,\vec{j} = 20\,\vec{j}$

$$\overrightarrow{OM}(0) = (\sqrt{3}\ \times 0\,)\vec{i} + \vec{C} = 20\,\vec{j}$$

$$\overrightarrow{OM}(t) = \sqrt{3}\ t\,\vec{i} + \vec{C} = \sqrt{3}\ t\,\vec{i} + 20\,\vec{j} \quad \textbf{(2 p)}$$

3. Calculate the **time required to cross the river**.

$$L = 60\ m = \sqrt{3}\ t \Rightarrow t = \frac{60}{\sqrt{3}} = 34.68\ s \quad \textbf{(1 p)}$$

4. Does the boat reach the point B? Justify your answer.

$$\overrightarrow{OM}(t) = \sqrt{3}\ t\,\vec{i} + 20\,\vec{j} \begin{cases} x(t) = \sqrt{3}\ t \\ y(t) = 20 \end{cases} \quad \textbf{(1 p)}$$

$y(t) = 20$ $\Rightarrow$ The boat will not arrive at point B, but rather at the point whose coordinates are (60, 20) **(1 p)**