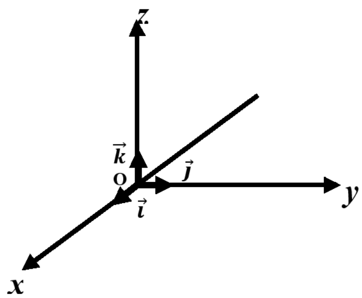
Exercise 01 (6 pts) :



A- Given the expressions of the vectors $\vec{A}$ and $\vec{B}$ in the Cartesian coordinates system as follows: $\vec{A} = 2\,\vec{i} + 2\,\vec{j} + 2\,\vec{k}$, $\vec{B} = 3\,\vec{i} + 3\,\vec{j} + 3\,\vec{k}$

1. Represent these two vectors in the provided coordinate system.

2. Calculate the dot product ($\vec{A}\,.\vec{B}$) and the cross product ($\vec{A} \wedge \vec{B}$) of the vectors, and deduce the angle between them

B- The mathematical expression for the quantity K is defined as: $K = \sqrt{2\,R\,g\,sin\,(\omega\,t)}$

Where R represents the radius, g the acceleration due to gravity, t denotes time, ω is a dimensionless constant.

1. Write the dimension for each : $[R]$, $[g]$, $[t]$, $[\omega]$, $[\sin(\omega\,t)]$

2. Derive the dimensional formula of $[K]$, then, Identify the physical nature of the quantity $[K]$.

Exercise 02 (6pts):

$(t^2, \pi\,t)$ are the coordinates of a moving object M in polar coordinates:

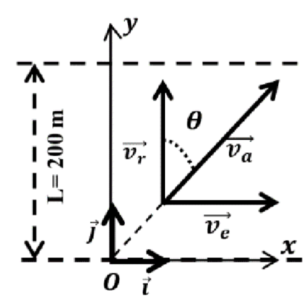
1. Derive the expressions for the position vector $\overrightarrow{OM}$, velocity $\vec{v}$, and acceleration $\vec{a}$ of the object M in the polar coordinates.

2. Convert the object's coordinates from polar $(t^2, \pi\,t)$ to Cartesian coordinates(x, y).

3. Write the expression for the position vector $\overrightarrow{OM}$ in Cartesian coordinates, and then find the displacement vector $\overrightarrow{M_0M_2}$ between the moments t_0=0 s and t_2 =2 s .

4. Calculate the average velocity $\overrightarrow{v_{0,2}}$ between these two moments $t_0 = 0$s and t_2 =2 s.

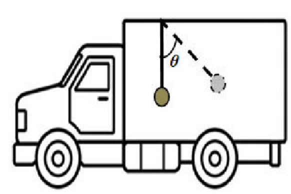
Exercise 03 (5 pts):



A rowboat is initially positioned perpendicular to the bank of a river with a width of L=200 m. The rower propels the boat at a velocity of 3 m/s with respect to the water. The river current has a velocity of 3 m/s.

1. **Write the analytical expressions for the relative velocity $\vec{v_r}$, the entrained velocity $\vec{v_e}$ and the absolute velocity $\vec{v_a}$ of the boat in the Cartesian coordinate system (Oxy).**
2. **Calculate the angle θ by which the boat deviates from its original direction.**
3. **Write the position vector $\overrightarrow{OM}$ (relative to point 0 on the riverbank).**
4. **Calculate the time required for the boat to reach the opposite bank.**

Exercise 04 (5 pts):

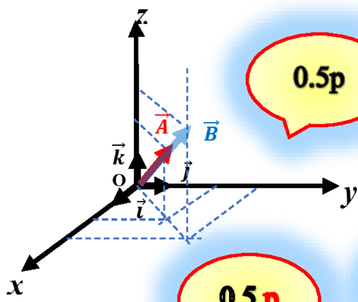


A small ball of mass m= g 20 is suspended by a massless thread from the roof of a truck.

1. **Represent the forces acting on the ball.**
2. **Calculate the angle θ formed by the thread when the truck accelerates at 2 m/s^2**
3. **Determine the tension in the thread.**

Exercise 01 (6 pts) :

A-1 Represent these two vectors in the provided coordinate system.



0.5p

0.5 p 1 p 0.5 p

1. $\vec{A}\cdot\vec{B} = 2\times 3 + 2\times 3 + 2\times 3 = 18$ $\quad \vec{A}\wedge\vec{B} = \vec{0}$ $\quad$ The angle between $\vec{A}$ *and* $\vec{B}$ $\theta = 0^o$

2.0 p

B-1 Write the dimension of: $[R] = L$ $\quad [g] = L\,T^{-2}$ $\quad [t] = T$ $\quad [\omega] = 1$ $\quad [\sin(\omega\, t)] = 1$

The dimension of K : $[K] = \sqrt{[2]\,[R]\,[g]\,[\sin(\omega\, t)]} = \sqrt{L\times L\,T^{-2}\times 1} = L\,T^{-1}$ 1 p

The physical nature of the quantity: $[K]$ $\Rightarrow$ velocity 0.5 p

Exercise 02 (6pts):

1. Position vector : $\overrightarrow{OM} = t^2\,\vec{u}_r$ 0.5 p

   Velocity vector: $\vec{v} = \frac{d\overrightarrow{OM}}{dt} = 2t\,\vec{u}_r + t^2\,\pi\,\vec{u}_\theta$ 1 p

   Acceleration vector $\vec{a} = \frac{d\vec{v}}{dt} = (2 - t^2)\,\vec{u}_r + 4t\,\pi\,\vec{u}_\theta$ 1.5 p

2. Converting the polar coordinates $(t^2, \pi\, t)$ to Cartesian coordinates(x, y).

$$(t^2, \pi\, t) \Rightarrow (x, y) \quad \begin{cases} x = t^2\,\cos(\pi\, t) \\ y = t^2\,\sin(\pi\, t) \end{cases}$$ 1 p

The position vector: $\overrightarrow{OM} = (t^2\,\cos(\pi\, t))\,\vec{\imath} + (t^2\,\sin(\pi\, t))\,\vec{\jmath}$ 1 p

The displacement vector: $\overrightarrow{M_0M_2} = \overrightarrow{OM_2} - \overrightarrow{OM_0} = 4\,\vec{\imath}$ where

$\overrightarrow{OM_0} = \overrightarrow{OM}\,(t = 0s) = \vec{0}$ $\quad \overrightarrow{OM_2} = \overrightarrow{OM}\,(t = 2s) = 4\,\vec{\imath}$

The average velocity: $\overrightarrow{v_{0,2}} = \frac{\overrightarrow{M_0M_2}}{t_2 - t_0} = 2\,\vec{\imath}$ 1 p

Exercise 03 (5 pts):

0.5 p 0.5 p 1 p

1. The analytical expressions :

$\overrightarrow{v_e} = 3\,\vec{\imath}$ $\quad \overrightarrow{v_r} = 3\,\vec{\jmath}$ $\quad \overrightarrow{v_a} = \overrightarrow{v_e} + \overrightarrow{v_r} = 3\,\vec{\imath} + 3\,\vec{\jmath} = v_a\sin\theta\,\vec{\imath} + v_a\cos\theta\,\vec{\jmath}$

2. The angle θ by which the boat deviates from its original direction.

$$\begin{cases} v_a \cos\alpha = 3 \\ v_a \sin\alpha = 3 \end{cases} \Rightarrow \tan\theta = \frac{3}{3} = 1 \Rightarrow \theta = 45^0$$

1 p

3. Write the position vector $\overrightarrow{OM}$ (relative to point 0 on the riverbank).

1 p

$$\vec{v_a} = 3\,\vec{\imath} + 3\,\vec{\jmath} = \frac{d(\overrightarrow{OM})}{dt}$$

$$d(\overrightarrow{OM}) = (3\,\vec{\imath} + 3\,\vec{\jmath})\ dt$$

$$\overrightarrow{OM} = \int (3\vec{\imath} + 3\,\vec{\jmath})\,dt = 3t\,\vec{\imath} + 3t\,\vec{\jmath}$$

4. Calculate the time required for the boat to reach the opposite bank

1 p

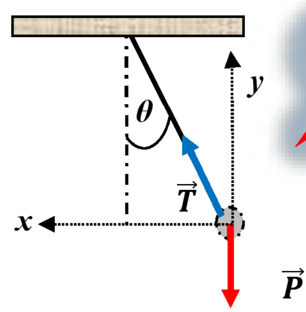
$$y = 3\,t \quad \Rightarrow \quad t = \frac{y}{3}$$

$$for\ y = L = 200\,m \Rightarrow t = \frac{200}{3} = 66.66\,s$$

Exercise 04 (5 pts):

1. Represent the forces acting on the ball.

1 p



2. Calculate the angle θ formed by the thread when the truck accelerates at 2 m/s^2

3 p

$$\sum \vec{F}_{ext} = m\,\vec{a} \Rightarrow \vec{P} + \vec{T} = m\,\vec{a}\ where \begin{cases} \vec{P} = -mg\,\vec{\jmath} \\ \vec{T} = T\,\sin\theta\,\vec{\imath} + T\,\cos\theta\,\vec{\jmath} \\ \vec{a} = a\,\vec{\imath} \end{cases}$$

$$-mg\,\vec{\jmath} + T\,\sin\theta\,\vec{\imath} + T\,\cos\theta\,\vec{\jmath} = m\,a\,\vec{\imath} \Rightarrow T\,\sin\theta\,\vec{\imath} + (T\,\cos\theta - mg)\,\vec{\jmath} = m\,a\,\vec{\imath}$$

$$\Rightarrow \begin{cases} T\,\sin\theta = ma \\ T\,\cos\theta - mg = 0 \end{cases} \Rightarrow \begin{cases} T\,\sin\theta = ma \\ T\,\cos\theta = mg \end{cases} \Rightarrow \frac{T\,\sin\theta}{T\,\cos\theta} = \frac{ma}{m\,g} \Rightarrow \tan\theta = \frac{a}{g} = \frac{2}{9.8} \Rightarrow \theta = 11.57^0$$

Determine the tension in the thread.

$$T\,\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta} = 0.2\,N$$

1 p