UNIVERSITY OF M'SILA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE
DEPARTMENT OF COMPUTER SCIENCE

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3^{rd} YEAR – COMPUTER ENGINEERING
SEMESTER 05

Exercise Series No. 1

Exercise 0.1. Let $(\Omega, \mathcal{P}(\Omega), p)$ be a probability space, and let A and B be two events with respective probabilities $p(A)$ and $p(B)$.

- Compute the probability $p[(A \cap \overline{B}) \cup (\overline{A} \cap B)]$ in terms of $p(A)$, $p(B)$, and $p(A \cap B)$.
- Suppose that $p(B) \neq 0$. Deduce from the previous question that $p(A/B) + p(\overline{A}/B) = 1$.

Exercise 0.2. The probability that an individual in a population is affected by a disease A is a given p; in the same population, an individual can also be affected by a disease B with a given probability q. We assume the two diseases are independent.

- What is the probability of being affected by both diseases?
- What is the probability of being affected by at least one of the two diseases?

Exercise 0.3. An urn contains x white balls and y red balls. Two balls are drawn simultaneously.

- What is the probability $p(x, y)$ that both balls are of the same color?
- What relationship must x and y satisfy so that $p(x, y) = \frac{1}{2}$? Which pairs (x, y) satisfy this condition?

Exercise 0.4. Two fair six-sided dice are rolled.

1. Give the sample space Ω of all possible outcomes and the associated probability p.
2. Find the probability of obtaining:
   - A double
   - At most one even number
   - Exactly one even number
   - Two consecutive numbers

Exercise 0.5. A factory has three machines A, B, and C producing the same type of mechanical part. Machine A produces 25% of the parts, B produces 35%, and C produces 40%. The percentage of defective parts is 5% for machine A, 4% for B, and 2% for C. A part is selected at random from the total production, and it is found to be defective. Compute the probability that it was produced:

1. By machine A.
2. By machine B.
3. By machine C.

Exercise 0.6. A player has 9 dice: two of type A, three of type B, and four of type C. The table below shows, for each type of die, the number of faces showing the number i $(i = 1, 2, 3, 4, 5, 6)$.

	1	2	3	4	5	6
A	1	2	1	0	1	1
B	1	2	2	0	1	0
C	2	2	2	0	0	0

The player randomly selects one die and rolls it three times, obtaining 4–2–1. Find the respective probabilities that the die used was of type A, B, or C.

Random Experiment – Event

- A **random experiment** is any experiment whose outcome is determined solely by chance.
- The set of all possible outcomes of a random experiment is called the **sample space**, denoted by Ω.
- $(\Omega, \mathcal{P}(\Omega))$ is called a (finite) measurable probability space.
- Any subset of Ω is called an **event**.
- An event containing a single element is called an **elementary event**.
- The empty set is called the **impossible event**.
- The event "A or B" is $A \cup B$ (either A or B occurs).
- The event "A and B" is $A \cap B$ (both A and B occur).
- The complement of A in Ω is denoted by $\overline{A}$.
- Events A and B are **incompatible** if $A \cap B = \varnothing$.
- A **complete system of events** in Ω is a finite family $(A_i)_i$ satisfying:

$$\bullet\ \forall i \neq j, A_i \cap A_j = \varnothing, \quad \bullet\ \bigcup_i A_i = \Omega$$

Finite Probability Space

A **probability** on Ω is any function $p : \mathcal{P}(\Omega) \to [0,1]$ satisfying:

$$\bullet\ p(\Omega) = 1, \quad \bullet\ \forall A, B \text{ disjoint}, p(A \cup B) = p(A) + p(B)$$

The pair (Ω, p) is called a **finite probability space**.

Conditional Probability

Let A and B be two random events such that $p(B) \neq 0$. The **conditional probability** of A given B is defined by:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \tag{1}$$

Law of Total Probability and Bayes' Theorem for n Events

- Let $(A_k)_{k \in K}$ be a complete system of random events in Ω:

$$P(A) = \sum_{k \in K} P(A/A_k)P(A_k). \tag{2}$$

- Let $(A_k)_{k \in K}$ be a complete system of random events in Ω. If for all $k \in K$, $P(A_k) > 0$, then for any event A with $P(A) > 0$, we have:

$$P(A_k/A) = \frac{P(A/A_k)P(A_k)}{\sum\limits_{i \in K} P(A/A_i)P(A_i)} \tag{3}$$