University of M'sila

Faculty of Mathematics and Computer Science
Department of Computer Science

Probability and Statistics
Model Correction with Inline Barème

Exercise 1 (5 points)

Let A and B be two events.

1. Compute $P[(A \cap \overline{B}) \cup (\overline{A} \cap B)]$ (3 pts)

The events $A \cap \overline{B}$ and $\overline{A} \cap B$ are disjoint. [0.5 pt]

$$P[(A \cap \overline{B}) \cup (\overline{A} \cap B)] = P(A \cap \overline{B}) + P(\overline{A} \cap B).$$

[0.5 pt]

$$P(A \cap \overline{B}) = P(A) - P(A \cap B), \quad P(\overline{A} \cap B) = P(B) - P(A \cap B).$$

[1 pt]

$$P[(A \cap \overline{B}) \cup (\overline{A} \cap B)] = P(A) + P(B) - 2P(A \cap B).$$

[1 pt]

2. Compute $P(A \mid \overline{B}) + P(\overline{A} \mid \overline{B})$ (2 pts)

$$P(A \mid \overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})}, \quad P(\overline{A} \mid \overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}.$$

[0.5 pt]

$$P(A \cap \overline{B}) + P(\overline{A} \cap \overline{B}) = P(\overline{B}).$$

[0.5 pt]

$$P(A \mid \overline{B}) + P(\overline{A} \mid \overline{B}) = 1.$$

[1 pt]

Exercise 2 (5 points)

An urn contains n white balls and m red balls.

1. Probability of same color **(2 pts)**

Two cases are possible: two white balls or two red balls. [0.5 pt]

$$P(\mathrm{WW}) = \frac{\binom{n}{2}}{\binom{n+m}{2}}, \quad P(\mathrm{RR}) = \frac{\binom{m}{2}}{\binom{n+m}{2}}.$$

[0.5 pt]

$$P = \frac{\binom{n}{2} + \binom{m}{2}}{\binom{n+m}{2}}.$$

[1 pt]

2. Condition for $P = \frac{1}{2}$ **(2 pts)**

$$\frac{\binom{n}{2} + \binom{m}{2}}{\binom{n+m}{2}} = \frac{1}{2}.$$

[0.5 pt]

After simplification,

$$(n-m)^2 = n + m.$$

[1.5 pt]

3. Integer solutions **(1 pt)**

All integer pairs (n, m) satisfying

$$(n-m)^2 = n + m$$

are solutions. [1 pt]

Exercise 3 (5 points)

Let X and Y be independent geometric random variables with parameter p.

$$P(X = k) = p(1-p)^{k-1}, \quad k \geq 1.$$

[1 pt]
[1 pt]

Since X and Y are independent,

$$P(X + Y = n) = \sum_{k=1}^{n-1} P(X = k)P(Y = n-k).$$

[1 pt]

$$P(X + Y = n) = (n-1)p^2(1-p)^{n-2}, \quad n \geq 2.$$

[2 pt]

Exercise 4 (5 points)

$$f(x) = \begin{cases} \frac{5}{4}(1-x)^{1/4}, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

1. Show that f is a density **(2 pts)**

$f(x) \geq 0$ on $[0, 1]$. [0.5 pt]

$$\int_0^1 \frac{5}{4}(1-x)^{1/4}\,dx = \frac{5}{4} \cdot \frac{4}{5} = 1.$$

[1.5 pt]

2. Cumulative distribution function **(2 pts)**

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - (1-x)^{5/4}, & 0 \leq x \leq 1, \\ 1, & x > 1. \end{cases}$$

[2 pt]

3. Compute $P(0.488 < X < 1.2)$ **(1 pt)**

$$P(0.488 < X < 1.2) = (0.512)^{5/4}.$$

[1 pt]