

Second-order linear PDE
Series of exercises 02

Exercise 01. Let us consider the PDE (Tricomi equation)

$$\frac{\partial^2 u(x, y)}{\partial x^2} + x \frac{\partial^2 u(x, y)}{\partial y^2} = 0, \quad (1)$$

- 1- Find the domain where this PDE is hyperbolic and elliptic.
- 2- In the domain where this PDE is hyperbolic:
 - a- Determine the characteristic curves.
 - b- Perform the corresponding change of coordinates to obtain the corresponding canonical form.

Exercise 02. Consider the following equation:

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0, \quad x \neq 0 \quad (2)$$

- 1- Determine the type of equation (2).
- 2- Find the characteristic curves.
- 3- Determine the canonical form of the equation (2).
- 4- Find the general solution to this equation.

Exercise 03. Consider the following equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} = 0 \quad (3)$$

- 1- Determine the type of equation (3).
- 2- Find the characteristic curves.
- 3- Determine the canonical form of the equation (3).

Exercise 04. Consider the following second-order partial differential equation:

$$\frac{\partial^2 u(x, y)}{\partial x^2} + y \frac{\partial^2 u(x, y)}{\partial y^2} = 0, \quad y > 0 \quad (4)$$

- 1- State the nature of the equation (4).
- 2- Determine the characteristic curves of the equation (4).
- 3- Find the canonical form of this equation.
4. Same questions for the equations:

a) $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} + 2 \frac{\partial^2 u}{\partial y^2} = 0.$

b) $\frac{\partial^2 u}{\partial x^2} + x^2 \frac{\partial^2 u}{\partial y^2} = 0.$

c) $x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} = 0.$

d) $x^2 \frac{\partial^2 u}{\partial x^2} - xy \frac{\partial^2 u}{\partial x \partial y} - 6y^2 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} = 0, \quad (x, y) \neq (0, 0)$