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TD3 : Numerical Solutions of First-Order ODEs

Exercise 1

Consider the Cauchy problem :

$$\begin{cases} y'(x) = -\frac{1}{1+x^2} y(x) \\ y(0) = 5 \end{cases}$$

Determine the value at $x = 2$ using the explicit Euler method and the implicit Euler method with a step size $h = 0.5$. Compare the results if the exact solution is $y = 5\, e^{-\arctan(x)}$.

Exercise 2

Consider the problem :

$$\begin{cases} y'(x) = xy(x) \\ y(1) = 2 \end{cases}$$

Compute the exact solution. Then compute the approximation of $y(2)$ using step sizes $h = 0.5,\ 0.25,\ 0.125$, with the second-order Taylor method.

Exercise 3

Consider the differential equation :

$$\begin{cases} y'(x) = y(x) - \dfrac{2x}{y(x)} \\ y(0) = 1 \end{cases}$$

Approximate the solution at $x = 0.4$ using :

- the second-order Runge-Kutta method with $h = 0.2$,
- the fourth-order Runge-Kutta method with $h = 0.2$.

Verify that $y(x) = \sqrt{2x+1}$ is the exact solution and deduce $y(0.4)$.

Exercise 4

Consider the Cauchy problem :

$$\begin{cases} y'(x) = -y(x) + e^{-x} + e^{x} \quad \text{for } x \in]0,1] \\ y(0) = \dfrac{1}{2} \end{cases}$$

Show that the exact solution is $y(x) = \dfrac{1}{2}e^{x} + xe^{-x}$. Then :

- Apply the explicit Euler method with $h = 0.1$, and compute $y(0.4)$ to 10^{-4} precision.
- Apply the modified Euler method with $h = 0.2$, and compute $y(0.4)$ to 10^{-4} precision.
- Compare both results to the exact value and comment.