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TD2 : Iterative Methods

Exercise 1 Consider the following system:

$$\begin{cases} 5x_1 + x_2 - 2x_3 = 15, \\ 3x_2 + x_3 = 7, \\ 3x_1 - 4x_2 + 8x_3 = 9. \end{cases}$$

1. Write the system in the matrix form $AX = b$ and give the matrices D, L, U such that $A = D - L - U$.
2. Determine the Jacobi matrix and show that $\rho(B_J) < 1$.
3. Calculate an approximation of the solution using the Jacobi method up to 10^{-1} accuracy.

Exercise 2 Consider the following system:

$$\begin{cases} 3x_1 + x_2 - x_3 = 2, \\ x_1 + 5x_2 + 2x_3 = 17, \\ 2x_1 - x_2 - 6x_3 = -18. \end{cases}$$

1. Write the system in matrix form $AX = b$ and give the matrices D, L, U such that $A = D - L - U$.
2. Determine the Gauss-Seidel matrix and show that $\rho(B_{GS}) < 1$.
3. Starting from $X^0 = (0.8, 2.1, 2.5)^t$, calculate an approximation of the solution using the Gauss-Seidel method with an accuracy of 10^{-1}.

Exercise 3 Given the matrix :

$$A = \begin{pmatrix} \alpha & 1 & 0 \\ 1 & \alpha & 1 \\ 0 & 1 & \alpha \end{pmatrix}, \quad \alpha \in \mathbb{R}$$

1. For which values of α is the matrix A invertible?
2. Write the Jacobi iteration matrix and deduce the necessary and sufficient condition on α for the Jacobi iterative method to converge.
3. Provide the Jacobi algorithm, then for $\alpha = 3$, compute the vector $X^{(1)}$ starting from

$$X^{(0)} = (0, 0, 0)^T \quad \text{and} \quad b = (1, 1, 1)^T.$$

4. Calculate the number of iterations required for the error to be less than 10^{-2}. Use the $\|\cdot\|_\infty$ norm.

Exercise 4 We consider the following matrix A:

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$$

1. Write the iteration matrix of the relaxation method and study the convergence of this method.
2. Find the optimal choice of w for the SOR method for the matrix.
3. Put$b = (1, 1, 1)^t$.Starting from $X^0 = (0, 0, 0)^t$, calculate an approximation of the solution with an accuracy of 10^{-1} and $w = 1.25$.