Mohamed Boudiaf university at m'sila
Faculty of Mathematics and Computer Science

Mathematics degree /L2/S4/ may **10, 2026**
Mathematics Department
Exam time: 2 hours

Final Exam Ana-Num 2

Exercise 1:(12pts)

Consider the following system:

$$\begin{cases} 6x_1 + x_2 + x_3 = 12 \\ 2x_1 + 4x_2 = 0 \\ x_1 + 2x_2 + 6x_3 = 6 \end{cases}$$

1. Write the system in matrix form $Ax = b$.
2. using Gauss Elimination solve the system.
3. Deduce the determinant of matrix A.
4. Show that A is strictly diagonally dominant.
5. give the matrices D, L and U such that A $= D - L - U$ and Calculate the B_{GS} matrix of the Gauss-Seidel method.
6. Calculate the spectral radius $\rho(B_{GS})$. Does Gauss-Seidel method converge?
7. Starting from $x^{(0)} = (2,2,2)^t$ calculate an approximation of the solution the Gauss-Seidel method with 3 iterations.
8. Starting from $x^{(0)} = (0,0,0)^t$ calculate an approximation of the solution the Jacobi method with 3 iterations.

Exercise 2:(8pts)

Consider the following Cauchy problem:

$$\begin{cases} y'(x) = -y(x) + x, & x \in [0,1] \\ y(0) = 1 \end{cases}$$

1. Show that problem has a unique solution.
2. Verify that $y = x - 1 + 2e^{-x}$ is the exact solution
3. Approximate the solution at $x = 0.3$ using the explicit Euler method for $h = 0.1$.
4. Compare the approximate value $y(0.3)$ with the exact solution.
5. Deduce an approximation of $y'(0.3)$. Then compare the result with the exact derivative.
6. Compute an approximate value of $y(0.2)$ using the Runge-Kutta method of order 4 with $h = 0.2$.
7. Compare the value $y(0.2)$ with the exact solution.
8. What is the error estimate for each method in terms of step h?

REMARK:

All results are four significant figures.