

Final Exam Ana-Num 2

Exercise 1:(12pts)

Consider the following system:

$$\begin{cases} 2x_1 - x_2 = 5 \\ -x_1 + 2x_2 - x_3 = -5 \\ -x_2 + 2x_3 = 3 \end{cases}$$

1. Write the system in matrix form $Ax = b$.
2. using Gauss Elimination solve the system.
3. Deduce the determinant of matrix A .
4. Show that A is symmetric positive definite.
5. give the matrices D, L and U such that $A = D - L - U$ and Calculate the B_J matrix of the Jacobi method.
6. Calculate the eigenvalues of B_J .
7. Calculate the spectral radius $\rho(B_J)$. Does Jacobi's method converge?
8. Starting from $x^{(0)} = (0,0,0)^t$ calculate an approximation of the solution the Jacobi method with 3 iterations.

Exercise 2:(8pts)

Consider the following Cauchy problem:

$$\begin{cases} y'(x) = -2x y(x), & x \in [0,1] \\ y(0) = 1 \end{cases}$$

1. Show that problem has a unique solution.
2. Approximate the solution for $h = 0.1$ at $x = 0.3$ using the explicit Euler method.
3. Deduce an approximation of $y'(0.3)$.
4. Compare the value $y(0.3)$ with the exact solution.
5. Compute an approximate value of $y(0.2)$ using the Runge-Kutta method of order 4 with $h = 0.2$.
6. Compare the value $y(0.2)$ with the exact solution.
7. What is the error estimate for each method in terms of step h ?

REMARKS:

1. The exact solution is: $y = e^{-x^2}$.
2. All results are four significant figures.