Numerical Analysis Exam

Academic Year: 2023−2024

Duration: 1.30 Hours **Total: 20 points**

Exercise 01 (10 points)

Let us consider the following system:

$$(*)\begin{cases} 4x_1 + 2x_2 + x_3 = 6 \\ 2x_1 + 5x_2 + x_3 = 10 \\ x_1 + x_2 + 3x_3 = -3 \end{cases}$$

1. Write the system $(*)$ in matrix form $Ax = b$.
2. Show that the matrix A admits a unique LU factorization.
3. Show that A is symmetric positive definite.
4. Perform the Cholesky factorization of A.
5. Use this factorization to solve the linear system $(*)$.
6. Verify that the iterative processes of Jacobi and Gauss-Seidel, associated with system $(*)$, converge for $x^{(0)} \in \mathbb{R}^3$.
7. Write the iteration system for the Jacobi and Gauss-Seidel methods.

Exercise 02 (10 points)

Consider the following Cauchy problem:

$$(*)\begin{cases} y'(t) = t + 1 - y(t), \quad t \in [0, 1] \\ y(0) = 1 \end{cases}$$

1. Verify that the exact solution of this problem is: $y(t) = e^{-t} + t$.
2. Approximate the solution of $(*)$ for $h = \frac{1}{10}$ at $t = 0.3$ using the explicit Euler method.
3. Deduce an approximation of $y'(0.3)$.
4. Compare the value $y(0.3)$ with the exact solution.
5. Compute an approximate value of $y(0.2)$ using the Runge-Kutta method of order 4 with step size $h = \frac{1}{5}$.
6. Compare the value $y(0.2)$ with the exact solution.