University of M'sila
Faculty of Mathematics and Computer Science
Department of Mathematics
2^{nd} Year Mathematics (2025-2026)

Series N°. 02 (Complex Analysis)

Note: Questions marked with $(*)$ are outside the TD session.

Exercice 1 *Graph the following regions in the complex plane:*
(a) $\{z : \mathrm{Re}(z) \geq 2\,\mathrm{Im}(z)\}$;
(b) $\{z : \frac{\pi}{2} \leq z \leq \frac{3\pi}{2}\}$;
(c) $\{z : |z - 4i + 2| > 2\}$.

Exercice 2 1) *Determine the order of all the zeros of the following functions:*
a) $f(z) = 1 - \cos(z)$, *b)*$^{(*)}$ $f(z) = z\sin(z)$, *c)* $f(z) = (1 - e^z)\sin(z)$, *d)*$^{(*)}$ $f(z) = z^3\sin(z^3)$.
2) *Compute the following integrals::*
a) $\int_\gamma (2\overline{z} + 5)\,dz$ *where* γ *is the segment* $[i + 1, 1]$.
b) $\int_\gamma y\,dz$ *where* γ *is the composition of the segments* $[0, i]$ *and* $[i, i + 2]$.
c)$^{(*)}$ $\int_\gamma (z^2 + 3z)\,dz$ *where* γ *is the straight segment joining the points* $(0; 0)$ *and* $(0; 1)$.
d) $\int_C \frac{dz}{z}$, $\int_C \frac{dz}{z^2}$, $\int_C \frac{dz}{|z|^2}$ *where* C *is the circle* $|z| = 1$.

Exercice 3 *Using Cauchy's integral formulas, compute the following integrals::*
a)$^{(*)}$ $\int_{|z|=4} \frac{ze^z}{(z-\pi)^2}$, *b)* $\int_{|z+2|=\frac{3}{2}} \frac{z^2+1}{z(z+1)}dz$, *c)* $\int_{|z|=5} \frac{dz}{z^2+16}$, *d)*$^{(*)}$ $\int_{|z|=1} \frac{\cos(z)}{z^3}dz$,
e) $\int_{|z-2|=3} \frac{e^z}{z^2-6z}dz$, *f)* $\int_{|z-i|=2} \frac{\cos(\pi z)}{(z^2+4)^2}dz$, *g)*$^{(*)}$ $\int_{C_k} \frac{e^{z^2}}{z^2-6}dz$ *where* C_k *are the following curves* $C_1 : |z - 2| = 1$, $C_2 : |z - 2| = 3$, $C_3 : |z - 2| = 5$.

Exercice 4 $^{(*)}$ *1) Expand the following functions into Laurent series around* z_0*, specifying the domains of convergence:*
a) $f(z) = \frac{1}{z(z-3)}$, $z_0 = 0$, *b)* $f(z) = \frac{e^z}{(z-1)^2}$, $z_0 = 1$, *c)*$^{(*)}$ $f(z) =$

$\dfrac{1}{(z+1)(z+2)}$, $z_0=-1$, $d)^{(*)}$ $f(z)=e^{\frac{1}{z}}$, $z_0=0$.

2) Find the Laurent series of the function

$$f(z)=\frac{1}{(z-2)(z-3)}$$

(a) in the annulus $2<|z|<3$,

(b) in the region $3<|z|<\infty$.

Exercice 5 $^{(*)}$*Find the number of roots of the following equations:*

(1) $z^4-5z+1=0$ inside the disk $|z|<\frac{1}{4}$

(2) $z^8-4z^5+z^2-1=0$ inside the disk $|z|<1$

(3) $z^{11}-5z^3+3z+60=0$ in the annulus $1<|z|<3$

(4) $3z^n-e^z=0$ in the disk $|z|<1$

Exercice 6 1) *Determine the singular points and specify their types, then compute the residues of the following functions:*

$a)$ $f(z)=\dfrac{e^z}{z^2(z-1)(z-2)}$, $b)$ $f(z)=\dfrac{e^z}{z}$, $c)^{(*)}$ $f(z)=\dfrac{1-\cos(z)}{z}$, $d)^{(*)}$ $f(z)=\dfrac{\log(1+z)}{z^2}$, $e)^{(*)}$ $f(z)=\dfrac{1-e^z}{1+e^z}$, $f)^{(*)}$ $f(z)=\dfrac{ze^z}{z^2-1}$, $g)^{(*)}$ $f(z)=e^{-\frac{1}{(z-1)^2}}$.

2) *Compute the following integrals::*

$\int_{|z|=2}\tan(z)\,dz$, $\int_{|z-i|=\frac{1}{2}}\dfrac{1}{z^4-1}dz$, $\int_{|z|=1}\sin\left(\dfrac{1}{z}\right)dz$, $\int_{|z-1|=3}\dfrac{e^z}{z(z+1)}dz$.

3) *Using the residue theorem, compute the following integrals: :*

$$\int_0^{2\pi}\frac{dx}{2+\cos(x)},\ \int_{-\infty}^{+\infty}\frac{dx}{x^4+1},\ \int_{-\infty}^{+\infty}\frac{dx}{x^6+1},\ \int_{-\infty}^{+\infty}\frac{dx}{(x^2+1)^2},$$

$$\int_{-\infty}^{+\infty}\frac{\sin(x)}{x}dx,\ \int_0^{+\infty}\frac{\cos(x)}{(x^2+1)^2}dx,\ \int_{-\pi}^{\pi}\frac{dx}{5+\cos x},\ \int_0^{+\infty}\frac{\sin(x)}{(x^2+1)x}dx,$$

$$\int_{-\infty}^{+\infty}\frac{e^{ix}}{x^2+x+1}dx,\ \int_{-\infty}^{+\infty}\frac{1}{x^\alpha(1+x)}dx\ et\ \alpha\in\,]0,1[\,,\ \int_0^{+\infty}\frac{\cos(x)}{\sqrt{x}(x^2+1)^2}dx,\ \int_{-\pi}^{\pi}\frac{dx}{2+\sin(x)}.$$