University of M'sila
Faculty of Mathematics and Computer Science
Department of Mathematics
2^{nd} year Mathematics(2025-2026)

Series N^o. 01 (Complex Analysis)

N.B: The questions $(*)$ are outside of the **T.D** sessions.

Exercice 1 *1) Write the following complex numbers in algebraic form:*
$z_1 = \frac{(1+i)^9}{(1-i)^7}$; $z_2 = \frac{1+\lambda i}{2\lambda + \left(\lambda^2 - 1\right) i}$, $\lambda \in \mathbb{R}$; $z_3 = (1+i)^{1000}$;
$z_4 = (\sqrt{3} - i)^3 \left(\frac{-1+i}{\sqrt{3}}\right)^{-5}$.
2) Write the following complex numbers in trigonometric form: $z_1 = 1 - i\sqrt{3}$ *and* $z_2 = \frac{1+i\sqrt{3}}{1-i}$.
3) Write the following complex numbers in exponential form: $a = 1 - i, b = -3, c = \sin x + i \cos x, x \in \mathbb{R}$.

Exercice 2 *Represent the following sets of points in the complex plane:*

1. $A = \{z \in \mathbb{C} : |z - 3i| \leq |z - 3|\}$, $B = \{z \in \mathbb{C} : |z - i| < 3\}$, $C = \{z \in \mathbb{C} : |z - i| > 3\}$, $D = \{z \in \mathbb{C} : \mathrm{Re}(z) - \mathrm{Im}(z) < 1\}$.

Exercice 3 *1) Calculate* $\sin(i), \cos(i), \log(i), \log(-1), Log(i), Log(-1)$.
2) Compare between $Log(i-1)^2$ *et* $2Log(i-1)$*, then conclude.*
3)$^{(*)}$ Prove the following identities, where $z = x + iy$ *with* $x, y \in \mathbb{R}$*:*
a) $|\sin z| = \sqrt{\cosh^2 y - \cos^2 x}$; *b)* $|\cos z| = \sqrt{\cosh^2 y - \sin^2 x}$;
c) $|\sinh z| = \sqrt{\cosh^2 x - \cos^2 y}$; *d)* $|\cosh z| = \sqrt{\cosh^2 x - \sin^2 y}$.
4) Solve the equations: $\sin(z) = 2$, $\exp(z) = -1$, $\exp(z) = i$, $z^5 - 1 = i$.

Exercice 4 *1) Find all the points of discontinuity of the following functions:*
a) $f(z) = \frac{z-2}{z^2 + 4z + 10}$; *b)$^{(*)}$* $f(z) = \frac{1}{z(z^2+1)}$; *c)$^{(*)}$* $f(z) = \frac{z^2+3}{z^3 - 27}$; *d)*

$f(z)=\dfrac{1}{z^4+1}$.

2)$^{(*)}$ Let f be a complex function given by:

$$f(z)=\frac{z^2-2iz-1}{z^4+2z^2+1}$$

a) Calculate $\lim\limits_{z\mapsto i} f(z)$.

b) f Is it continuous at $z=i$?

c) f Can it be extended by continuity at $z=i$?

Exercice 5 *1) Let $G=P+IQ$, Express the functions P, Q in terms of x and y in the following cases:*

a) $G=z^3$; *b)* $G=z+\dfrac{1}{z}$; *c)$^{(*)}$* $G=\dfrac{z}{z+1}$; *d)$^{(*)}$* $G=ze^z$.

2) Find a domain Ω on which the following functions are holomorphic, and then find the expression of f' in terms of z:

a) $f(z)=z^3$; *b)* $f(z)=\sin(z)$; *c)$^{(*)}$* $f(x+iy)=\dfrac{x-iy}{(x^2+y^2)^2}$; *d)* $f(x+iy)=x^2+y^2$; *e)* $f(x+iy)=x^2+y^2-2ixy$; *f)$^{(*)}$* $f(z)=\left(z-\frac{1}{z}\right)^3$; *g)$^{(*)}$* $f(z)=\dfrac{z^2-2}{z^2-3z-2}$.

3)$^{(*)}$ Express the Cauchy-Riemann conditions in terms of polar coordinates.

Exercice 6 *a) Show that the function P is the real part of a holomorphic function in the following cases:*

1) $P(x,y)=x^3+6x^2y-3xy^2-2y^3$ *et* $f(0)=i$; *2)* $P(x,y)=x^3-3xy^2$; *3)* $P(r,\theta)=r^3\cos(3\theta)$; *4)$^{(*)}$* $P(x,y)=\arctan\left(\dfrac{y}{x}\right)$.

b)$^{(*)}$ Soit $P(x,y)=x^3-3\lambda xy^2+2x+1$. *Give a necessary and sufficient condition on λ for P to be the real part of a holomorphic function f on $\mathbb{C}$ and find f.*

c) Is the mean value theorem valid for holomorphic functions?