University of M'sila

Faculty: Maths-Compt-Science

Multiples integrals

L2 Maths

Module: Analysis 4

Exercise series n°5

Bachelor's degree in mathematics-L2 2025

Remark 1

The exercise marked by ($\star$) or supplementary will not be corrected in the sience of **TD**.

Exercise 1

Calculate the following integrals on the indicated domains

1. $\displaystyle\iint_D \frac{dxdy}{(1+x)(1+y)}, \quad D=[0,1]\times[0,1].$
2. $\displaystyle\iint_D \frac{y}{x^2+y^2}dxdy, \quad D=[0,1]\times[1,2].$
3. $\displaystyle\iint_D \frac{x\sin y}{1+x^2}dxdy, \quad D=[0,1]\times[0,\pi]$ ($\star$).
4. $\displaystyle\iint_D xydxdy, \quad D=\{(x,y)\in\mathbb{R}^2:\ x>0,\ y>0, \text{ and } x^2+y^2<1\}.$
5. $\displaystyle\iint_D x\cos ydxdy, \quad D=\{(x,y)\in\mathbb{R}^2:\ 0<x<1, \text{ and } x^2<y<x\}$ ($\star$).

Exercise 2

Calculate the double integral $I=\displaystyle\iint_K xy^2dxdy$, on K such that

$$K=\{(x,y)\in\mathbb{R}^2:\ x+y\geq 1,\ x^2+y^2\leq 1\}.$$

Exercise 3

Plot(Draw) the domains of integration and compute the following double integrals, changing the order of integration if necessary.

1. $\displaystyle I_1=\int_{-1}^{2}\int_{x^2-1}^{x+1}(x^2+y)dxdy.$
2. $\displaystyle I_2=\int_{0}^{4}\int_{\frac{y}{2}}^{\sqrt{y}}(xy+y^3)dxdy$ ($\star$).
3. $\displaystyle I_3=\int_{1}^{10}\int_{0}^{\frac{1}{y}}ye^{xy}dxdy.$

Exercise 4

Calculate the triple integral of the function $f(x,y,z)=xyz$
in the volume bounded by the planes $x=0,\ y=0,\ z=0$ and $x+y+z-1=0$.

Exercise 5

Compute the double integral $I=\displaystyle\iint_D (x+y)^2e^{x^2-y^2}dxdy$

where the integration domain D is given by $D = \{(x,y) \in \mathbb{R}^2 : \ x \geq 0, \ y \geq 0 \text{ and } x + y \leq 1\}$, using the change of the following variables $u = x + y$ and $v = x - y$.

Exercise 6 ☆☆★

Using the change of variables in polar coordinates, calculate the following double integrals

1. $I_1 = \displaystyle\iint_{D_1} \frac{dxdy}{1 + x^2 + y^2}, \quad D_1 = \{(x,y) \in \mathbb{R}^2 : \ x^2 + y^2 \leq 1\}.$

2. $I_2 = \displaystyle\iint_{D_2} e^{-(x^2+y^2)} dxdy \quad D_2 = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1, \ \ 0 \leq y \leq \sqrt{1 - x^2}\} \ (\star).$

Exercise 7 ☆★★

Calculate the double integral $I = \displaystyle\iint_{D} \frac{\cos(x^2 + y^2)}{2 + \sin(x^2 + y^2)} dxdy$

where the integration domain D is defined by $D = \{(x,y) \in \mathbb{R}^2 : \ 1 \leq x^2 + y^2 \leq 4\}$.

Supplementary exercise 1 ☆★★

Draw(Illustrate) the integration domains and calculate the following double integrals

1. $I_1 = \displaystyle\iint_{D_1} (x \sin y) dxdy, \quad D_1 = \{(x,y) \in \mathbb{R}^2 : \ 0 \leq x \leq \pi, \ 0 \leq y \leq x\}.$

2. $I_2 = \displaystyle\iint_{D_2} 3y^3 e^{xy} dxdy, \quad D_2 = \{(x,y) \in \mathbb{R}^2 : \ 0 \leq x \leq 1, \ \sqrt{x} \leq y \leq 1\}.$

3. $I_4 = \displaystyle\iint_{D_4} (x^2 + y^2) dxdy, \quad D_4 = \{(x,y) \in \mathbb{R}^2 : \ x \geq 2, \ y \geq 2, \text{ and } x + y \leq 1\}.$

Supplementary exercise 2 ☆☆★

Let the triple integral $I = \displaystyle\iiint_{D} \frac{z^3 dxdydz}{(y+z)(x+y+z)}$, where D is the domain defined by

$$D_3 = \{(x,y,z) \in \mathbb{R}^3 : \ x \geq 0, \ \ y \geq 0, \ z \geq 0, \text{and } x + y + z \leq 1 \geq 0, \leq 4 - x^2\},$$

using the change of variables $u = x + y + z, \ v = y + z$ and $w = z$.

Exercise series n°5

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