Extrema

Remark 1

The exercise marked by ($\star$) or supplementary will not be corrected in the sience of **TD**.

Exercise 1

Consider the application f defined by

$$\begin{array}{rcl} f:\mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x,y) & \longmapsto & x^2+y^3-3y. \end{array}$$

1. Prove that f is differentiable on $\mathbb{R}^2$ and give it's gradient at any point $(x,y)\in\mathbb{R}^2$.
2. Determine the critical points of the application f.
3. Study the possible extrema of f, by specifying their nature (local minimum, global minimum, local maximum, global maximum, or no local extremum, hence no global extremum).

Exercise 2

Let f be a function of $\mathbb{R}^2$ in $\mathbb{R}$ defined by $f(x,y)=x^2+2xy+y-y^3$.

1. Determine the critical points of f.
2. State (Give) their nature (local extremum, saddle point, etc.)
3. Show that the local minimum obtained is not a global minimum for f.

Exercise 3

Let the function $f:(x,y)\longmapsto\ x\ln^2 x+y^2$

1. Specify the definition domain for f.
2. (a) Find the critical points of f. | (b) Determine their nature.
3. (a) Show that the local minimum obtained is in fact a global minimum. | (b) Does f have a global maximum ?

Exercise 4

Study the extrema of f under the constraint $g(x,y)=0$ in the following cases,

1. $f(x,y)=2x^2+2xy+y^2-6x-2y-3$ and $g(x,y)=x+y-4$
2. $f(x,y)=10\sqrt[4]{x}+2\sqrt[3]{y}$ and $g(x,y)=\dfrac{1}{4}\ln x+\dfrac{1}{3}\ln y-3$ ($\star$).

Exercise 5

Let the function

$$\begin{array}{rcl} f:\mathbb{R}_+^3 & \longrightarrow & \mathbb{R} \\ (x,y,z) & \longmapsto & x+y+z. \end{array}$$

We will study the maximum of the function f under the constraint $x^2+y^2+z^2=1$

1. Explain why this maximum exists.
2. Write the Lagrangian associated with the problem.
3. Then determine the point at which the maximum is achieved and the value of that maximum.

Exercise 6

Let the equation,

$$x^5+xyz+y^3+3xz^4=2 \qquad (1)$$

1. Show that the equation (1) defines in the neighborhood of the point $(1,-1)$ an implicit function $z=g(x,y)$, such that $g(1,-1)=1$.
2. Give the equation of the plane tangent to the surface $z=g(x,y)$ at point $(1,-1)$.

Supplementary exercise 1

Let the function

$$\begin{array}{rcl} f:\mathbb{R}^2 & \longrightarrow & \mathbb{R} \\ (x,y) & \longmapsto & \dfrac{x^2}{9}+\dfrac{y^2}{4}, \end{array}$$

where the variables x and y are related by the constraint $x^2+y=1$.

1. Determine the critical points of the Lagrangian $L(x,y,\lambda)$ associated with the problem.
2. Using the constraint equation, express f as a function of only the variable x.
3. Study(Examine) the function obtained and deduce the nature of the critical points.
4. Does the function f admit a global maximum under the constraint ?

Supplementary exercise 2

Which point on the circle with equation $y=4-\sqrt{3}x^2+y^2=1$ minimizes the distance to the line $y=4-\sqrt{3}x$.

Supplementary exercise 3

Find the extremums of the function $f(x,y,z)=3x-y-3z$ under the constraints constraints $x+y-z=0$ and $x^2+2z^2=6$.