University of M'sila	**Interrogation**	**Faculty : Mathematics-Computer Sc**
L2 Mathematics	**With correction**	**Academic year : 2025/2026**
Module : Analysis 4		**Duration : 45 minute**

Scale	**Exercise : 1** (5pts)
	Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by : $f(x, y) = \ln(x^2 + y^2)$.
1.5	(1) Determine and represent the level Line of f.
1.5	(2) Let the point $a(1, 2)$ and the vetor $\vec{v} = (3, -4)$. Calculate $D_{\vec{v}} f(1, 2)$ the directional derivative of f at the point a along the vector $\vec{v}$.
2	(3) Calculate df_a the differential of f at the point $a(1, 2)$.

Scale	**Exercise : 2** (3pts)
	Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by : $f(x, y) = \begin{cases} y^2 \ln \lvert x \rvert & : x \neq 0 \\ 0 & : \text{otherwise.} \end{cases}$
1	(1) Let the point $(e^{-\frac{1}{y^2}}, y)$, $y \neq 0$. Calculate $\lim\limits_{y \to 0} f(e^{-\frac{1}{y^2}}, y)$ and $\lim\limits_{y \to +\infty} f(e^{-\frac{1}{y^2}}, y)$.
1	(2) Is f continuous at $(0, 0)$?
1	(3) Is f differentiable at $(0, 0)$?
End	**12 avril 2026** — **Good luck**

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Scale	**Correction of exercise 1:** 5pt
1	(1) We determine and drawing the level Line of f. Let $k \in \mathbb{R}$, then, we have :
	$k = f(x,y) \Leftrightarrow k = \ln(x^2+y^2) \Leftrightarrow x^2+y^2 = e^k.$
0.5	Therefore, the level Line of f are the circle of center $O(0,0)$ and raduis $R = e^k$
1 + 0.5	(2) The directionl derivative at $a(1,2)$ along $\vec{v}$. Using L'Hospital theorem, we obtain :
	$D_{\vec{v}}f(1,2) = \lim\limits_{t\to 0}\frac{f(1+3t,2-4t)-f(1,2)}{t} = \lim\limits_{t\to 0}\frac{\ln(5t^2-2t+1)}{t} = \lim\limits_{t\to 0}\frac{10t-2}{5t^2-2t+1} = -2.$
	(3) L'application differential of f at $a(1,2)$. For all $(h,k) \in \mathbb{R}$ we have :
0.5	$df_a(h,k) = h\partial_x f(a) + k\partial_y f(a)$. Since
0.5 × 2	$\partial_x f(x,y) = \frac{2x}{x^2+y^2} \Rightarrow \partial_x f(a) = \frac{2}{5}$ $\partial_y f(x,y) = \frac{2y}{x^2+y^2} \Rightarrow \partial_y f(a) = \frac{4}{5}.$
0.5	Then, $df_a(h,k) = \frac{2}{5}h + \frac{4}{5}k.$

Scale	**Correction of exercise 2:** 3pt
	Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by : $f(x,y) = \begin{cases} y^2 \ln\lvert x\rvert & : x \neq 0 \\ 0 & : \text{otherwise.} \end{cases}$
	(1) We calcule the limits $\lim\limits_{y\to 0} f(e^{-\frac{1}{y^2}}, y)$ and $\lim\limits_{y\to +\infty} f(e^{-\frac{1}{y^2}}, y)$.
0.5 × 2	$\lim\limits_{y\to 0} f(e^{-\frac{1}{y^2}}, y) = \lim\limits_{y\to 0} y^2 \ln e^{-\frac{1}{y^2}} = \lim\limits_{y\to 0}(-1) = -1 = \lim\limits_{y\to +\infty} f(e^{-\frac{1}{y^2}}, y).$
1	(2) According to previous question f is not continuous at $(0,0)$, because :
	$\lim\limits_{y\to 0} f(e^{-\frac{1}{y^2}}, y) \neq 0 = f(0,0).$
1	(3) We know that differentiability implies continuity, therefore, according to the contrapositive, f is not continuous at $(0,0)$.
End	**12 avril 2026** **Good luck**