

University of M'sila	 Final Exam	Faculty : Mathematics-Computer Sc
L2 Mathematics		Academic year : 2025/2026
Module : Analysis 4		Duration : 2 hours

Scale	Exercise : 1 
2	Let $f : \mathbb{R}^2 \setminus \{(1, 0)\} \rightarrow \mathbb{R}$ be the function defined by : $f(x, y) = \frac{(x-1)^2 y}{(x-1)^2 + y^2}$.  Show that the function f can be extended by continuity at $(1, 0)$.
1	 Define $\tilde{f}$, its extended function at $(1, 0)$.
Scale	Exercise : 2 
1 + 1	Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined by : $f(x, y) = \begin{cases} \frac{x^3 y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$  Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ on $\mathbb{R}^2 \setminus \{(0, 0)\}$.
1 + 1	 Show that f has partial derivatives at $(0, 0)$ and calculate them.
1.5	 Is f differentiable at $(0, 0)$?
1.5	 Is f of class C^1 on $\mathbb{R}^2$?
Scale	Exercise 3: 
1.5	Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined by : $f(x, y) = x^3 + y^3 - 3xy$.  Determine the critical points of the function f.
2.5	 Determine the nature of these critical points (local maximum, local minimum, or saddle point) using the Hessian matrix.
2	 Does the function f has a global maximum or minimum on $\mathbb{R}^2$?
Scale	Exercise : 4 
0.5	Let $f : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined by : $f(x, y) = xy$, where D is the following domain : $D = \{(x, y) \in \mathbb{R}^2 : x \geq 0, 0 \leq y \leq x(4-x)\}$.  Draw the domain D in an orthonormal coordinate system $(o, \vec{i}, \vec{j})$.
1.5	 Determine $\mu(D)$, the area of D.
2	 Compute the double integral $I = \iint_D (xy) dx dy$.
End	Dr. Bouafia Dahmane Good luck