Series 01: Mathematical Logic
2025/2026

University of M'sila
Second Year Mathematics

Exercise 01

Using proof by contradiction, show that the two lines D and D' with equations $y = x + 1$ and $y = x - 1$, respectively, are parallel.

Use only the fact that in $\mathbb{R}^2$, two non-parallel lines intersect.

Exercise 02

Using proof by cases, show the following assertion:

$$\forall n \in \mathbb{N},\ \exists k \in \mathbb{N},\ n^2 = 4k \quad \text{or} \quad n^2 = 4k + 1.$$

Exercise 03

Using mathematical induction, prove the following assertion:

$$\forall n \in \mathbb{N}^*,\ 7 \mid \left(3^{2n} - 2^n\right).$$

Exercise 04

Let $a \in \mathbb{R}$. Using proof by contraposition, show that:

$$(\forall \varepsilon > 0,\ |a| < \varepsilon) \Rightarrow a = 0.$$