

TD4 : Numerical Differentiation and Integration

Exercise 1:

Let f a function defined by $f(x) = \ln x$. We choose the step size $h = 0.5$ and $h = 0.01$.

1. Approximate $f'(1.5)$ using the forward difference formula, backward difference formula, and the centered difference formula.
2. Calculate the relative error in each case.
3. Approximate $f''(1.5)$ using the centered difference formula, and determine bounds for the approximation errors.

Exercise 2:

Approximate $\int_0^1 x^2 e^{-x^2} dx$ using $h = 0.25$. Use

1. Composite Trapezoidal rule.
2. Composite Simpson's rule.

Exercise 3:

Determine the values of h and n required to approximate

$$\int_0^2 \frac{1}{x+4} dx$$

to within 10^{-5} and compute the approximation. Use

1. Composite Trapezoidal rule.
2. Composite Simpson's rule.
3. Compare the above methods with exact integration for $h = 0.5$ and $h = 0.25$. Comment on the results.