Mohamed Boudiaf university at m'sila **Mathematics Department**
Faculty of Mathematics and Computer Science **LMD-S3-Année 2024-2025**

TD2 : Solution of Nonlinear Equations in One Variable

Exercise 1:

Let the following equation:

$$f(x) = x^3 + 2x - 1$$

1. Justify that $\boldsymbol{f(x) = 0}$ has a unique solution $\boldsymbol{\alpha}$ in $\mathbb{R}$
2. Verify that this equation admits a unique root α in the interval [0,1].
3. Determine the minimum number of iterations required to calculate an approximate root of α with a precision $\epsilon = 10^{-1}$, using the bisection method, and then calculate this approximate root.
4. Perform two iterations with Newton's method starting from $x_0 = 1$.

Exercise 2:

We aim to find the root of $f(x) = 0$ using the fixed-point iteration method, where:

$$f(x) = e^x - x - 2$$

1. Show that $f(x) = 0$ has a unique solution in the interval [1,2].

Let $g(x) = ln(x + 2)$ be a function defined on the interval [1,2].

2. How was the function g obtained?

3. Is the fixed-point iteration method convergent?

4. Perform two iterations starting from $x_0 = 1$.

5. Determine the number of iterations needed for the error to be equal to 10^{-2}.

6. Apply Newton's method to the equation $f(x) = 0$ and perform two iterations starting from $x_0 = 2$.

Exercise 3:

Given the equation $x^3 + 4x^2 = 10$ in [1,2].

Show that the previous equation admits a unique solution in [1.2].

How many steps are required to calculate this solution using the bisection method with a precision of 10^{-4}?

Given the following iterative formulas: $x_{n+1} = \sqrt{(\frac{10}{x_n} - 4x_n)}$, $x_{n+1} = \sqrt{\frac{10}{x_n+4}}$; $n \geq 0$.

Which method converges to this solution? Why?

Write the Newton's iterative formula. What is its order of convergence? The root is approximately $\alpha = 1.3652$.