

Final Exam Ana-Num 1

Exercice1:(6pts)

Let $f(x) = 1 + \cos(\pi x)$ and the points $x_0 = 0, x_1 = \frac{1}{4}, x_2 = \frac{1}{2}, x_3 = \frac{3}{4}, x_4 = 1$.

1. Determine the Newton's interpolation polynomial of f at these points.
2. Deduce the approximate values of $f(1/3)$ Compare the results obtained.
3. Calculate the interpolation error and compare it with what you found in step 2.
4. Approximate $f'(1/4)$ using the forward difference formula, backward difference formula, and the centered difference formula. Calculate the relative error in the last case.
5. Compare the results to the derivative of Newton's interpolation polynomial.

Exercice2:(8pts)

Given the following function:

$$f(x) = \exp(x) - 3x^2$$

1. Justify that $f(x) = 0$ admits a unique solution α in the interval $[3,4]$.

Consider the fixed-point method

$$\begin{cases} x_{n+1} = g(x_n), n = 0,1,2 \dots \\ x_0 \in [3,4] \text{ and } g(x) = \ln(3x^2) \end{cases}$$

2. Examine the convergence of this method and determine its order of convergence.
 3. In this method, Determine the number of iterations needed for the error to be equal to 10^{-2} .
 4. Calculate an approximate root of α with a precision $\epsilon = 0.01$ for Newton's method and the fixed-point method where $x_0 = 4$.
 5. Between Newton's method and the fixed-point method, which is more efficient? Justify your answer.
- (The root is approximately $\alpha \approx 3.7331$)

Exercice3:(6pts)

We are looking for an approximation of the integral:

$$I = \int_0^1 \frac{1}{x^2 + 1} dx$$

Find the approximate value of I by:

- 1- The simple trapezoidal method
- 2- Simpson's simple method
- 3- The composite trapezoidal method with $n = 4$ (number of subintervals).
- 4- How many subdivisions of $[0, 1]$ are needed to evaluate I to within 10^{-2} accuracy using the composite trapezoidal method?
- 5- Calculate the absolute and relative error, exact value and approximate value in question 3.

$$\text{we give, } \max_{0 \leq x \leq 1} |f''(x)| = \frac{1}{2}, \text{ where } f(x) = \frac{1}{x^2 + 1}$$

REMARK: (All results are four significant figures)