

Final Exam Ana-Num 1

Exercise1:(6pts)

Let $f(x) = \sin \pi x$ and the points $x_0 = -1, x_1 = \frac{1}{2}, x_2 = 1, x_3 = \frac{3}{2}$.

1. Determine the Newton interpolation polynomial of f at these points.
2. Deduce the approximate values of $f(1/4)$ Compare the results obtained.
3. Calculate the interpolation error and compare it with what you found in step 2.

Exercise2:(8pts)

Given the following function:

$$f(x) = e^x - 4x$$

1. Study the variations of the function f on the interval $[0, +\infty[$.
2. Show that there is a root α between $[0, 1]$
3. How many iterations are needed to achieve an approximation with an accuracy of 0.08 in the Bisection method?

Consider the fixed-point method

$$\begin{cases} x_{n+1} = g(x_n), n = 0, 1, 2 \dots \dots \\ x_0 \in [0, 1] \text{ and } g(x) = \frac{e^x}{4} \end{cases}$$

4. Examine the convergence of this method and determine its order of convergence.
5. Write the Newton's method for finding the zeros of the function f .
6. calculate an approximate root of α with a precision $\epsilon = 0.001$ for Newton's method and the fixed-point method.
7. Between Newton's method and the fixed-point method, which is more efficient? Justify your answer.

Exercise3:(6pts)

$$I = \int_0^1 \sin \pi x \, dx$$

provide an approximate value of I , using:

1. The simple trapezoidal method.
2. The simple Simpson's method.
3. The composite trapezoidal method with $n = 4$ (number of subintervals).
4. How many subdivisions of $[0,1]$ are needed to evaluate I to within 10^{-5} accuracy using the composite trapezoidal method?
5. Calculate the absolute and relative error, exact value and approximate value in question 3.

REMARK: (All results are four significant figures)