تصحيح امتحان تحليل عددي 1 2026-2025

التمرين الأول : 7 نقاط

1. Newton's method .... (~~[illegible]~~)

x_i	$f(x_i)$				
0	2	-1.1716	-3.3137	4.4183	0
0.25	1.7071	-2.8284	0	4.4183	
0.5	1	-2.8284	3.3137		
0.75	0.2929	-1.1716			
1	0				
	0.5	0.25	0.25	0.25	

$$P(x) = 2 - 1.1716x - 3.3137x(x-0.25) + 4.4183x(x-0.25)(x-0.5) \text{ --- } 0.5$$

$$p(x) = 4.4183x^3 - 6.6274x^2 + 0.2091x + 2$$

2. $f(1/3) = 1.5$, $p(1/3) = 1.4970$ --- 0.5

$$e = |f(1/3) - p(1/3)| = 0.0030 \text{ --- } 0.5$$

3. interpolation error

$$E = |f(x) - p(x)| = \frac{|f^{(5)}(c)|}{5!}\left|(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)\overset{(x-x_5)}{\cancel{(x-x_5)}}\right|, \ c \in [0,1] \quad 0.5$$

$$f^{(5)}(c) = \pi^5 \sin(\pi c) \qquad |f^{5}(c)| \leq \pi^5 \quad 0.5$$

$$E = |f(1/3) - p(1/3)| \leq \frac{\pi^5}{120}\left|(1/3)(1/3 - 1/4)(1/3 - 1/2)(1/3 - 3/4)(1/3 - 1)\right| \leq 0.0033 \text{ --- } 0.5$$

4. $f'_g(\frac{1}{4}) = -1.1716$ (0.25) , $f'_d(\frac{1}{4}) = -2.8284$ (0.25) , $f'_c(\frac{1}{4}) = -2$ (0.25) , $f'(\frac{1}{4}) = -2.2214$ (0.25)

$E_A = |f'_c(\frac{1}{4}) - f'(\frac{1}{4})| = 0.2214$ (0.25) $\qquad E_R = \frac{E_A}{|f'(1/4)|} = \cancel{0.0997}\ 0.0997$ (0.25)

5. $p'(1/4) = \cancel{\ldots} -2.2761$, $|f'(1/4) - p'(1/4)| = 0.0547$ (0.5)

التمرين الثاني : 7 نقاط

1. f increasing, continuous on $[3,4]$ and $f(3) \cdot f(4) < 0$ ---- (1)

2. According (I.V.T) the equation $f(x) = 0$ has one solution in $[3,4]$.

2. Study of convergence of the sequence. 1

$g([3,4]) = [g(3), g(4)] = [3.2958, 3.8712] \subset [3,4]$

$g'(x) = \frac{2}{x}$, $\max_{3 \le x \le 4} |g'(x)| = \frac{2}{3} < 1$

then there is exactly one fixed-point in $[3,4]$

the convergence method is linear (first-order) 0.5

3. the iteration in the method where $\varepsilon = 10^{-2}$

$|x_n - \alpha| \le \frac{k^n}{1-k} |x_1 - x_0|$, $x_0 = 4$, $x_1 = \ln(48) \simeq 3.8712$

$\frac{(\frac{2}{3})^n}{1-2/3} |x_1 - x_0| \le \varepsilon \Rightarrow (\frac{2}{3})^n \le \frac{\varepsilon}{3|x_1 - x_0|}$ 1

$n \ge \ln\left(\frac{\varepsilon}{3|x_1 - x_0|}\right) / \ln(2/3)$, $n \geq 6.3$, $n \simeq 7$.

4. Newton's method.

$$\begin{cases} x_0 = 4 \\ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2 \ldots \end{cases}$$ 0.5

$f'(x) = e^x - 6x$ 0.5

Fixed point-method	Newton's method
$x_0 = 4$	$x_0 = 4$
$x_1 = \ln(3 \cdot x_0^2) = 3.8057$	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3.7844$
$x_2 = \ln(3x_1^2) = 3.7716$	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.7354$
$x_3 = \ln(3x_2^2) = 3.7536$	$\|x_2 - x_1\| = 0.0490$
~~$x_4 = \ln(3 x_3^2) = 3.7441$~~	$x_2 = 3.7354$
$\|x_3 - x_2\| = 0.0180$	1
$x_3 = 3.7536$ 1	

Newton's method is more efficient than method because it converges faster, and its order of convergence is 2 while the fixed-point method has order 1 0.5

التمرين الثالث : 6 نقاط

1. Simple trapezoidal method

$a=0,\ b=1,\ f(x)=\frac{1}{x^2+1}$

$$I=\int_a^b f(x)\,dx \simeq \frac{b-a}{2}\left(f(a)+f(b)\right)=0.75$$ 0,5

2. Simple simpson's method

$$I=\int_a^b f(x)\,dx=\frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)=0.7833$$ 1

3. Composite trapezoidal method $(n=4)$

$h=\frac{b-a}{4}=\frac{1}{4}$ 0,5 , $x_0=0,\ x_1=\frac{1}{4},\ x_2=1/2,\ x_3=3/4,\ x_4=1$

$$I=\int_a^b f(x)\,dx=\frac{h}{2}\left(f(a)+f(b)+2\left(f(x_1)+f(x_2)+f(x_3)\right)\right)=0.7828$$ 1

4. $$|R_t(f)|=\frac{(b-a)^3}{12n^2}|f^{(2)}(c)|\leq\frac{1}{12n^2}\max_{0\leq x\leq 1}|f^{(2)}(c)|\leq\frac{1}{24n^2}$$ 1

$$\frac{1}{24n^2}\leq 10^{-2}\iff n\geq 4.1667,\ \text{so } n\simeq 5$$ 0,5

exact value $I=\int_0^1\frac{1}{1+x^2}\,dx=\arctan(x)\Big|_0^1=\frac{\pi}{4}\simeq 0.7854$ 0,5

Absolute error $E_A=|0.7854-0.7828|=0.0026$ 0,5

Relative error: $E_R=\frac{0.0026}{0.7854}=0.0033$ 0,5