M'sila University	Series n° : 5 **Improper integrals**	Faculty of Mathematics and Computer Science
L 2 Mathematics		Module: Analysis 3

Remark 1

The exercise marked with $(\star)$ or additional will not be corrected in the session of **TD**.

Exercise 1

Determine if the following integrals are convergent and, if so, calculate their values

1. $\int_0^{+\infty} e^{-x}dx$.
2. $\int_0^1 \frac{1}{\sqrt{1-x}}dx$.
3. $\int_{-\infty}^0 xe^{-x^2}dx$.
4. $\int_0^{+\infty} xe^{-x}dx$.
5. $\int_0^{+\infty} \frac{1}{1+x^2}dx$.

Exercise 2

Compute the following generalized integrals

1. $\int_1^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$.
2. $\int_0^{\frac{\pi}{2}} \frac{\cos 2x}{\sqrt{\sqrt{\sin 2x}}}dx$ $(\star)$.
3. $\int_0^{+\infty} \frac{\arctan(x)}{1+x^2}dx$.
4. $\int_1^{+\infty} \frac{\ln x}{x}dx$.

Exercise 3

Examine the nature of the following integrals

1. $\int_0^{+\infty} \sin x dx$ $(\star)$.
2. $\int_1^{+\infty} \sin(x^2)dx$.
3. $\int_0^{+\infty} \frac{\cos(x^2)}{1+x^2}dx$ $(\star)$.
4. $\int_1^{+\infty} \frac{\ln x}{x+2}dx$.
5. $\int_0^{+\infty} \frac{\cos x}{x^2}dx$ $(\star)$.

Exercise 4

Let F be the function defined by $F(x) = \int_1^x \frac{\ln(1+t^2)}{t^2}dt$.

1. Compute $F(x)$.
2. Deduce the convergence of the following integral, then determine it's value,

$$I = \int_1^{+\infty} \frac{\ln(1+t^2)}{t^2}dt.$$

Exercise 5

Show that the following integrals are semi-convergent.

1. $\int_{\pi}^{+\infty} \frac{\cos x}{\sqrt{x}} dx.$
2. $\int_{-1}^{+\infty} \cos(x^2) dx, \ (\text{put } u = x^2).$

Supplementary exercise 1

Show that the integral $\int_{0}^{\frac{\pi}{2}} \tan x dx$ diverges,

1. Using a primitive calculation.
2. By the Riemann criterion.

Supplementary exercise 2

Study the nature of integrals

1. $\int_{0}^{+\infty} \frac{\sin(5x) - \sin(3x)}{x^{\frac{5}{3}}}.$
2. $\int_{0}^{+\infty} \frac{x}{e^x - 1}.$

Supplementary exercise 3

1. Study for $\alpha > 0$, the convergence and absolute convergence of the integral $\int_{1}^{+\infty} \frac{\sin(x)}{x^{\alpha}} dx.$
2. Depending on the values of the real parameters α and β, study the nature of the integral,

$$\int_{0}^{1} \frac{x^{\alpha}}{\ln^{\beta}(x+1)} dx.$$

Supplementary exercise 4

1. Study for which values of $n \in \mathbb{N}$ the integral $I_n = \int_{0}^{+\infty} \frac{1}{(x^3+1)^n} dx$ converges
2. Calculate I_1.
3. Show that for all $n \geq 2$, we have $I_{n+1} = \frac{3n-1}{3n} I_n.$
4. We deduce I_n if $n \geq 1$.