University of M'sila	Final exam	Faculty : Mathematics-Computer Sc
Level : L2− Mathematics		Academic year : 2025/2026
Module : Analysis 3	Third semester	Duration : 2 h

Scale	**Exercise : 1** 4pts
	Let the numerical series $\sum_{n\geq 0} u_n$, and S its sum, where $u_n = \frac{2}{(n+1)(n+3)}$.
1.5	① Use the equivelent theorem to show that the series $\sum_{n\geq 0} u_n$ is convergent.
0.5	② Check that $\forall n \in \mathbb{N}:\ u_n = \left(\frac{1}{n+1} - \frac{1}{n+2}\right) + \left(\frac{1}{n+2} - \frac{1}{n+3}\right)$.
2	③ Simplify the partial sum $S_n = \sum_{k=0}^{k=n} u_k$, and then deduce the value of the sum S.
	Exercise : 2 4pts
	Let the sequence of functions $(f_n)_{n\geq 1}$ be defined on $\mathbb{R}$ by $f_n(x) = \sqrt{x^2 + \frac{1}{n}}$.
1.5	① Show that the sequence $(f_n)_{n\geq 1}$ simply converges on $\mathbb{R}$ to the function $f:\ x \mapsto \lvert x \rvert$.
1	② Conclude that the sequence $(f_n)_{n\geq 1}$ is absolutely convergent on $\mathbb{R}$.
1.5	③ Study the uniform convergence of the sequence $(f_n)_{n\geq 1}$ on $\mathbb{R}$.
	Exercise : 3 6pts
	Let the function series $\sum_{n\geq 1} (-1)^n f_n$ be defined on $\mathbb{R}_+$ where $f_n(x) = \ln\left(1 + \frac{1}{n(1+x)}\right)$.
3	① Show that the series $\sum_{n\geq 1} (-1)^n f_n$ is uniformly convergent on $\mathbb{R}_+$.
3	② Prove that the series $\sum_{n\geq 1} (-1)^n f_n$ is not normally convergent on $\mathbb{R}_+$.
	An indication : $\forall n \in \mathbb{N}^*,\ \forall x \in \mathbb{R}_+:\ 1 + \frac{1}{(n+1)(1+x)} \leq 1 + \frac{1}{n(1+x)} \leq 1 + \frac{1}{n}$
	Exercice : 4 6pts
	Let be the integer series $\sum_{n\geq 0} \frac{n+2}{n+1} x^n$ and S its sum.
2	① Find R the radius and D the domain of convergence for this series.
2	② Calculate the sum S.
2	③ Deduce $\bar{S}$ and $\hat{S}$ the sums of following integers series $\sum_{n=0}^{+\infty} \frac{n-1}{n+1} x^n$ and $\sum_{n=0}^{+\infty} \frac{n+3}{n+1} x^n$.
End	**18 janvier 2026** — **Good luck**