University of M'sila	Final Exam	Faculty : Mathematics-Computer Sc
Level : L2– Mathematics		Academic year : 2024/2025
Module : Analysis 3	Third semester	Duration : 1 h 30 m

Scale	**Exercise : 1** (5pts)
	Study the nature of the following numerical series :
1	(1) $\sum_{n\geq 1}\left(1+\frac{1}{n}\right)^n$.
2	(2) $\sum_{n\geq 1}\ln\left(1+\frac{1}{n}\right)$.
2	(3) $\sum_{n\geq 0}\frac{e^{-n}}{n+1}$.
	Indication : $\forall n\in\mathbb{N}^*:\ \ln\left(1+\frac{1}{n}\right)=\ln(1+n)-\ln n$.

Scale	**Exercise : 2** (8pts)
	I Let the sequence of functions $(f_n)_{n\geq 0}$ be defined on $\mathbb{R}_+$ by $f_n(x)=e^{-n(x+1)}$.
2	(1) Show that the sequence $(f_n)_{n\geq 0}$ is simply converges on $\mathbb{R}_+$.
2	(2) Show that the sequence $(f_n)_{n\geq 0}$ is uniformly converges on $\mathbb{R}_+$.
	II Let the function series $\sum_{n\geq 0}e^{-n(x+1)}$ and S its sum on $\mathbb{R}_+$.
2	(1) Proof that the series $\sum_{n\geq 0}e^{-n(x+1)}$ is normally convergent on $\mathbb{R}_+$.
2	(2) Calculate the sum S.

Scale	**Exercice : 3** (7pts)
	Let be the integer series $\sum_{n\geq 0}\frac{1}{n+1}x^n$ and S its sum.
2.5	(1) Find R the radius and D the domain of convergence for this series.
2.5	(2) Calculate the sum S.
2	(3) Deduce the value of the sum $\sum_{n=0}^{+\infty}\frac{(-1)^n}{n+1}$.

End	**18 janvier 2025**	**Good luck**