Graphs Theory
Trees and Forest(4)

University of M’sila
Faculty of Mathematics and informatics
Mathematics department

Speciality:

Applied Mathematics

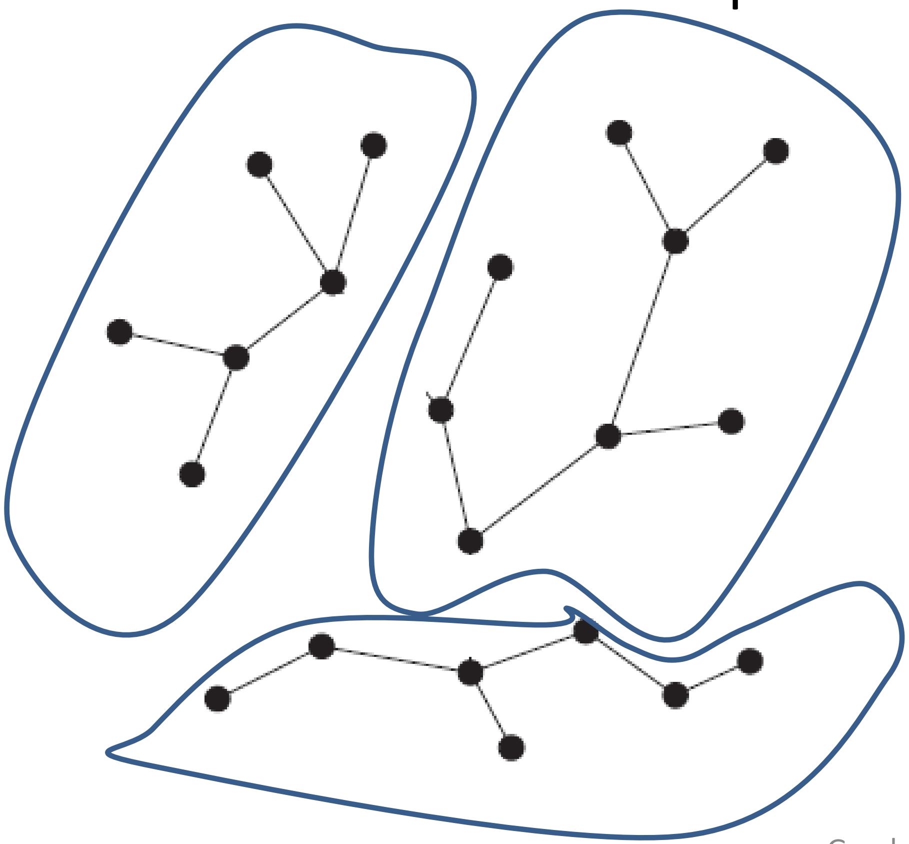
L3

2024/2025

MOHAMED BAHACHE

Tree and Forest

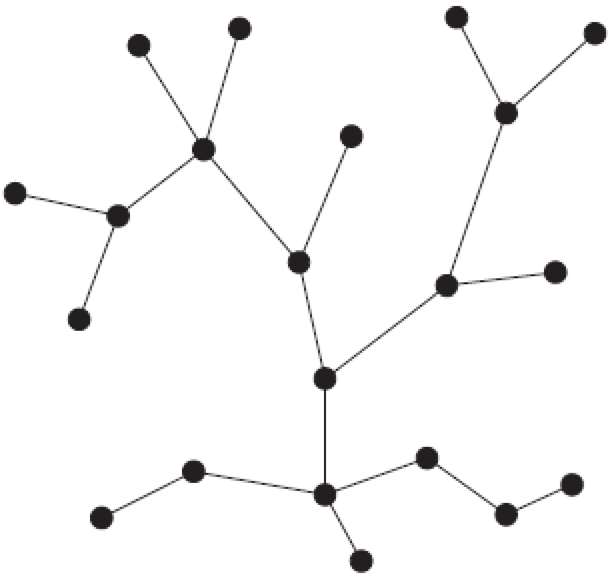
- An acyclic graph (not containing any cycle) is called a forest
- Connected forest is called tree
- Then a forest whose components are trees.



Forest

Tree

- Definition : a connected acyclic graph is called tree



Tree

Tree

- Properties :
  - The vertices of degree 1 in a tree are called leaves.
  - Remove a leaf from a tree it still tree.

Theorem : G is a graph the following statements are equivalents :

(1). G is a tree;

(2). Any two vertices from G are linked by a unique path;

(3). G is minimally connected (G-e is disconnected for every edge $e \in G$

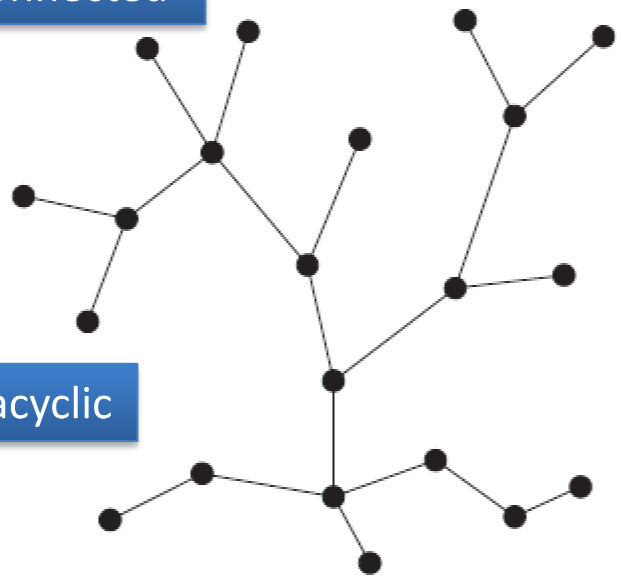
(4). G is maximally acyclic.(G+uv is cyclic for any two non adjacent vertices)

Tree

❑Example:

(1). G is a tree;

(3).G is minimally connected

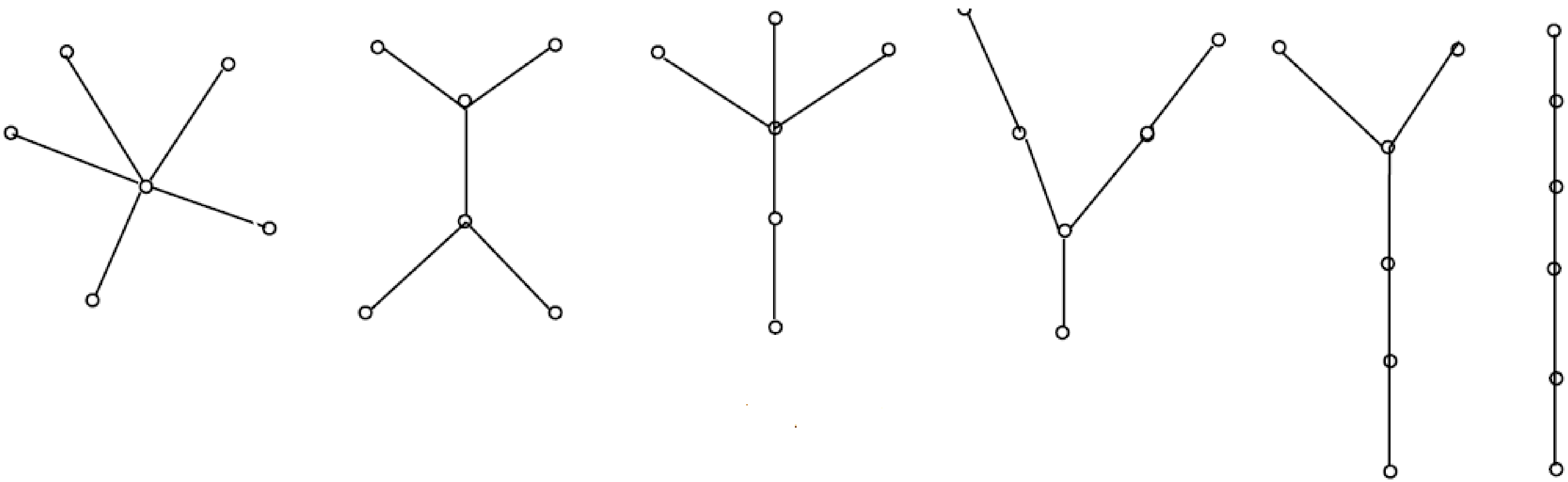


(4).G is maximally acyclic

(2). Any two vertices from G are linked by a unique path;

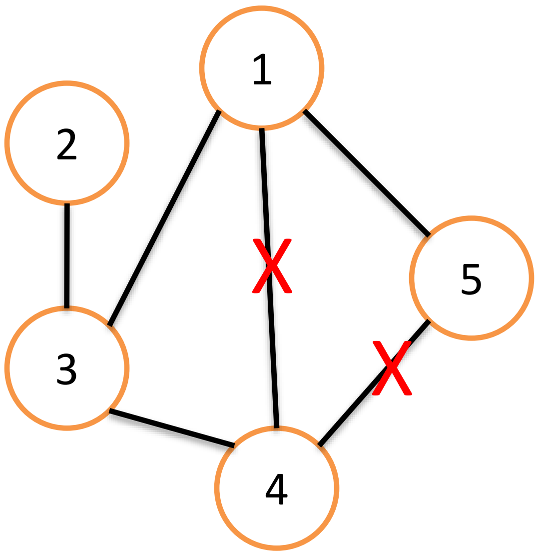
Tree

Corollary: Every non – trivial tree G has at least two vertices of degree 1.

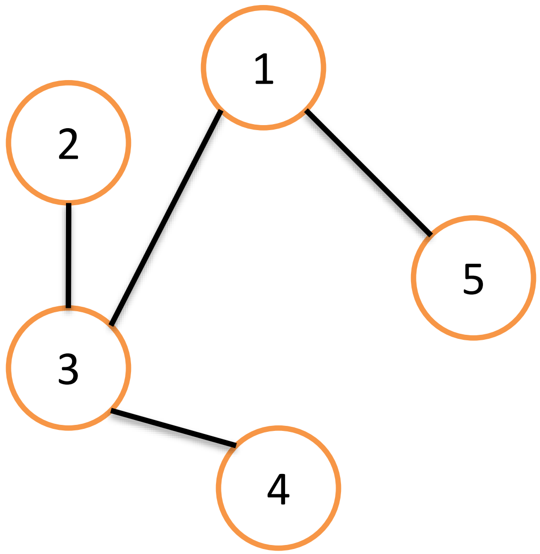


Spanning Tree

Theorem: Every connected graph contains a spanning tree

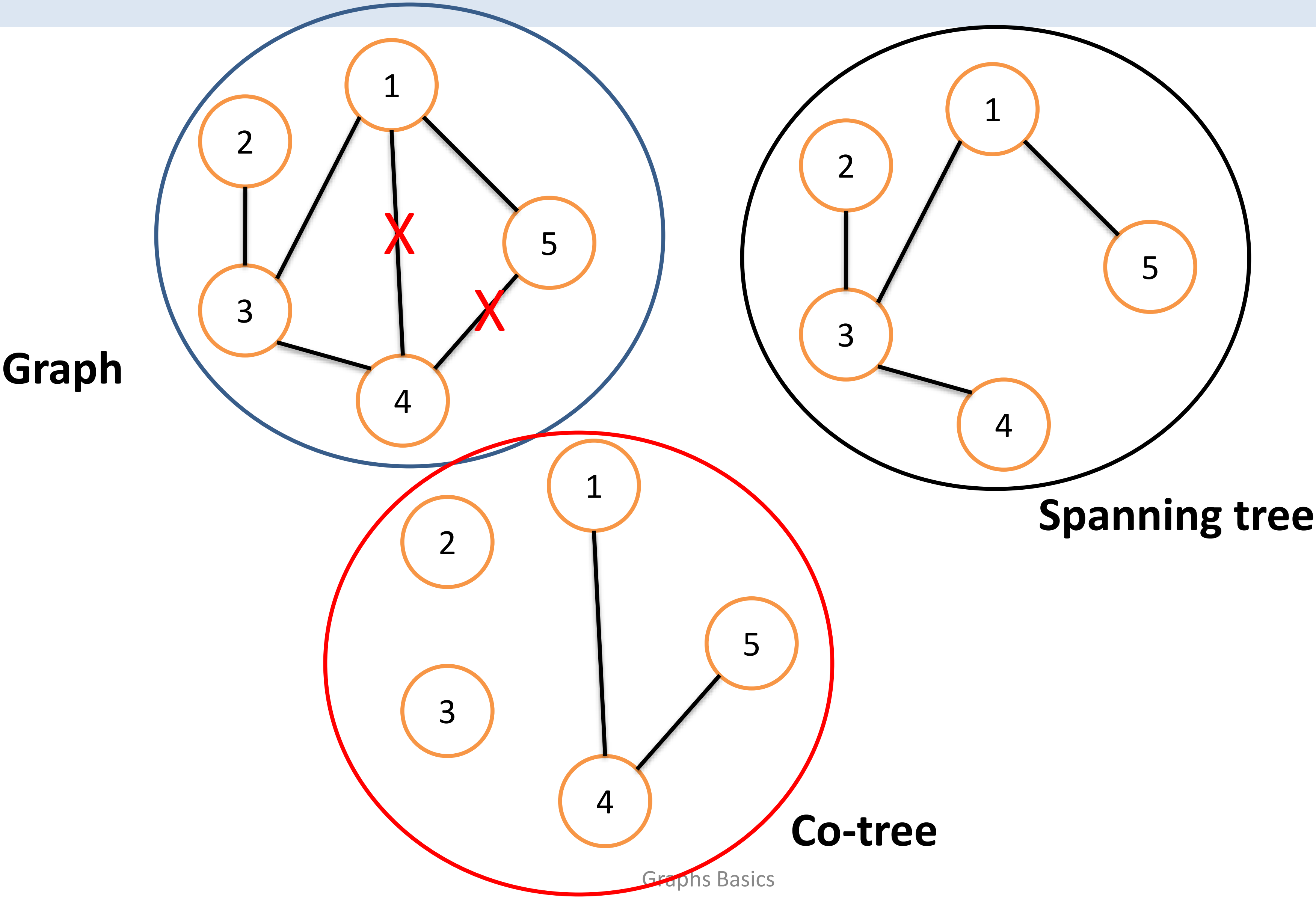


Graph



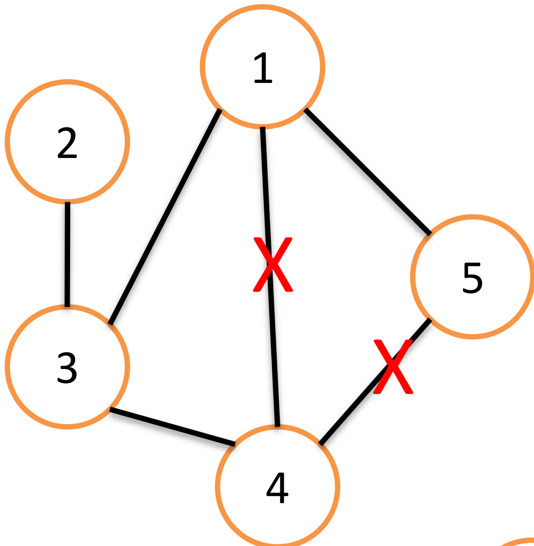
Spanning tree

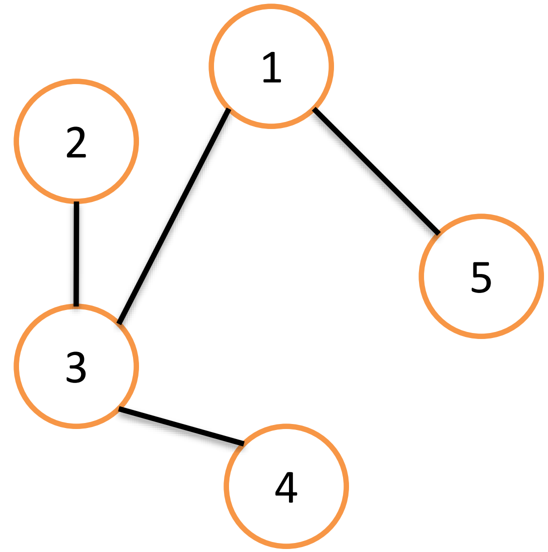
Spanning Tree



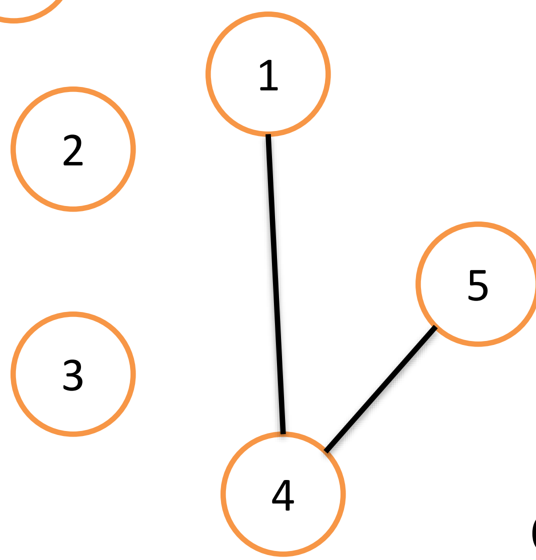
Spanning Tree

Graph





Spanning tree
Partial graph
G1(V,E1), $E1 \subset E$



Co-tree , partial graph G2(V,E2) $E2 \subset E$

Minimum Spanning Tree (MST)

- When working on a connected graph, we need to transform this graph into a tree to solve certain problems.
- Here , there are several trees.
- When our connected graph is a valuated graph , we need to determine the tree with (minimum/maximum) weight (MST).
- Prim's Algorithm (local approach , in each stage among the connected vertices , we chose the optimal edge to link between linked vertex and not linked vertex).
- Kruskal's Algorithm(Global approach we choose the optimal edge so that the added are does not connect not already connected vertices.)

Minimum Spanning Tree (MST)

❑ **Prim's Algorithm-** Local approach

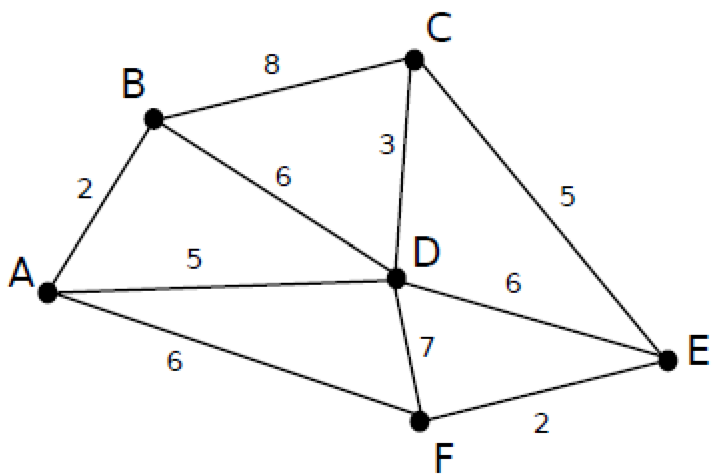
1-Choose any starting vertex. Look at all edges connecting to the vertex and choose the one with the lowest weight and add this to the tree.

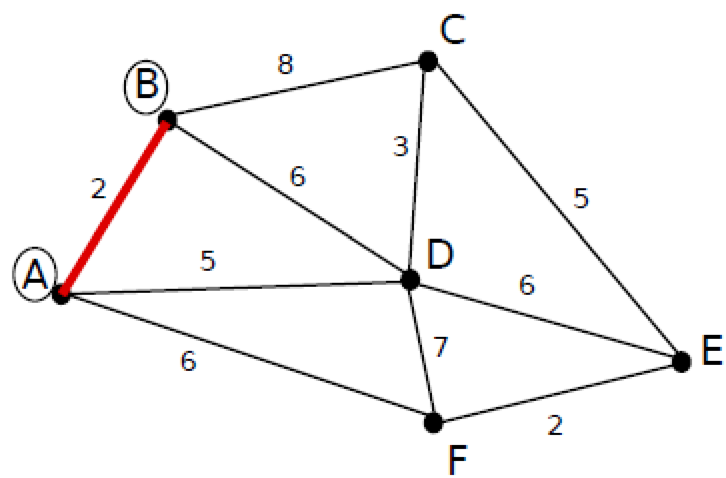
2- Look at all edges connected to the tree that do not have both vertices in the tree. Choose the one with the lowest weight and add it to the tree.

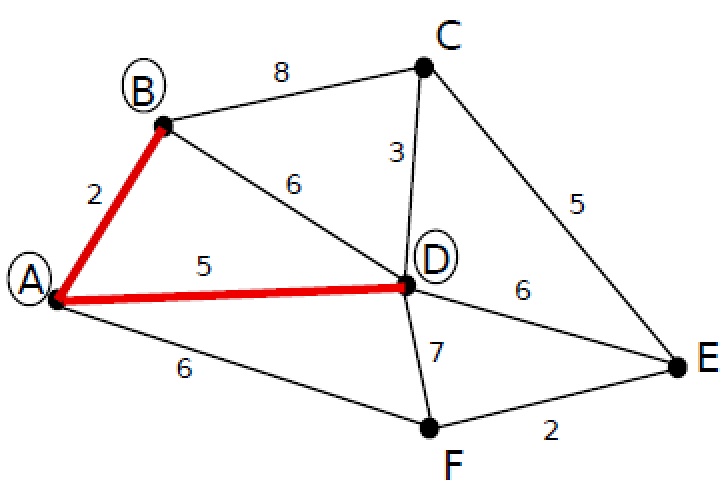
3-Repeat step 2 until all vertices are in the tree.

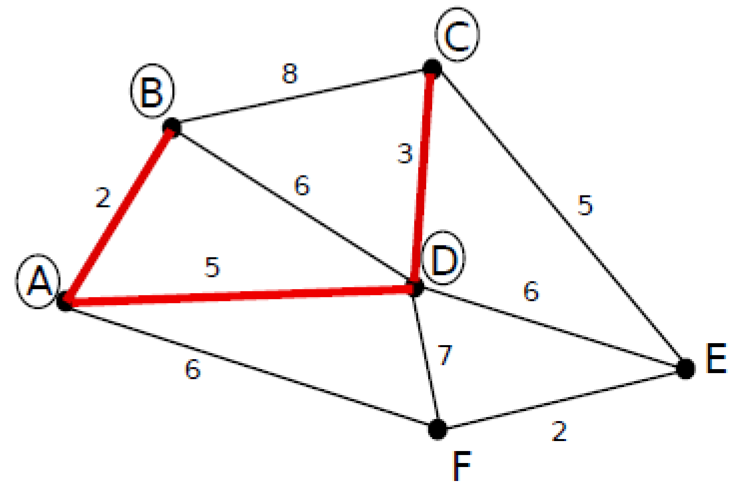
Remark: its complexity is $O(V^2)$

Minimum Spanning Tree (MST)

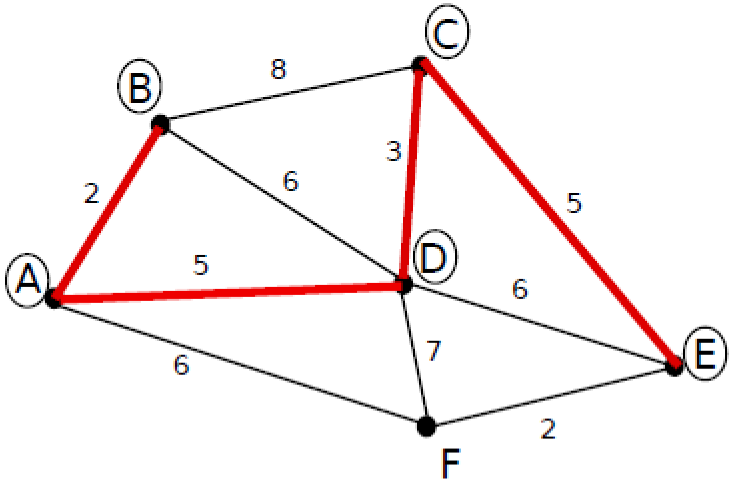


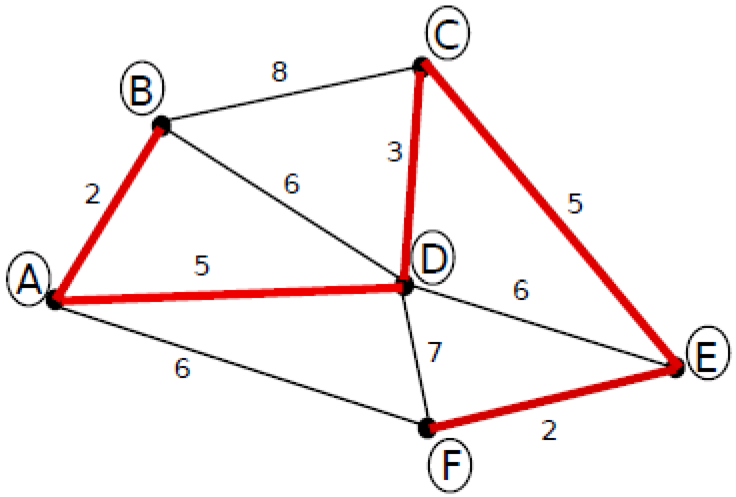






Minimum Spanning Tree (MST)





Minimum Spanning Tree (MST)

❑**Kruskal's Algorithm-** Global approach

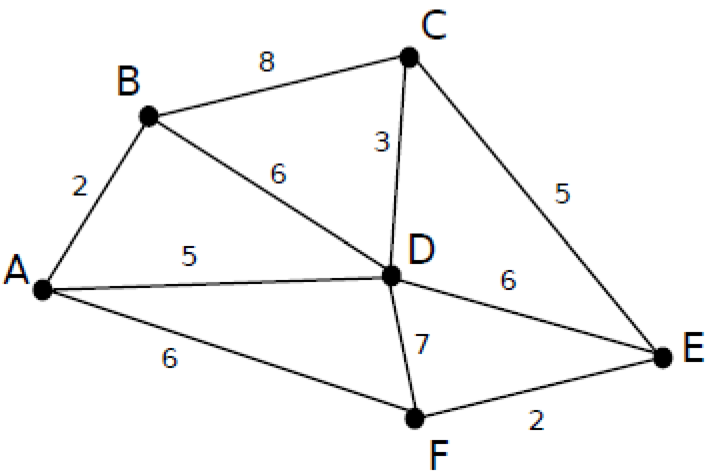
1-Sort all the edges in increase order.

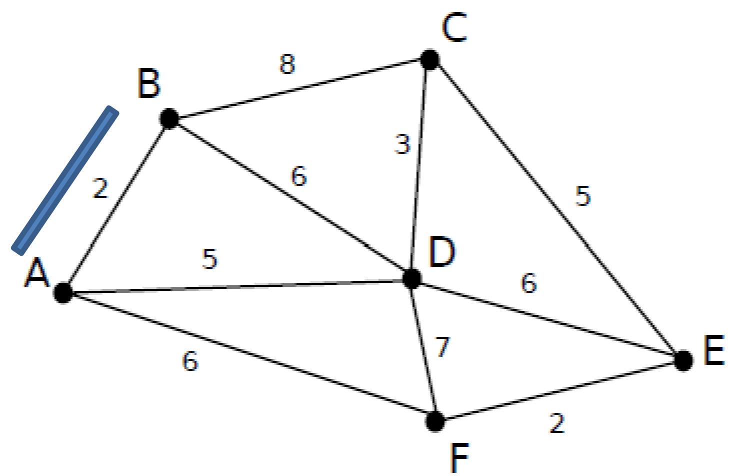
2- Take the edge with the lowest weight and add it to the spanning tree , if the added edge creates a cycle then this edge should be removed.

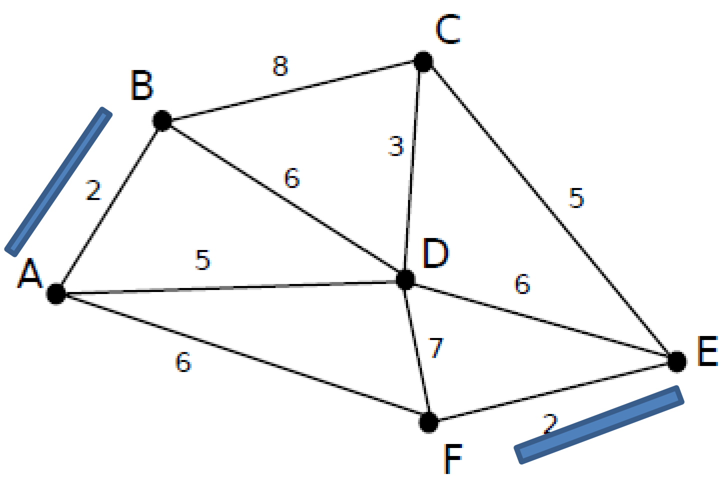
3-Repeat step 2 until reach the tree.

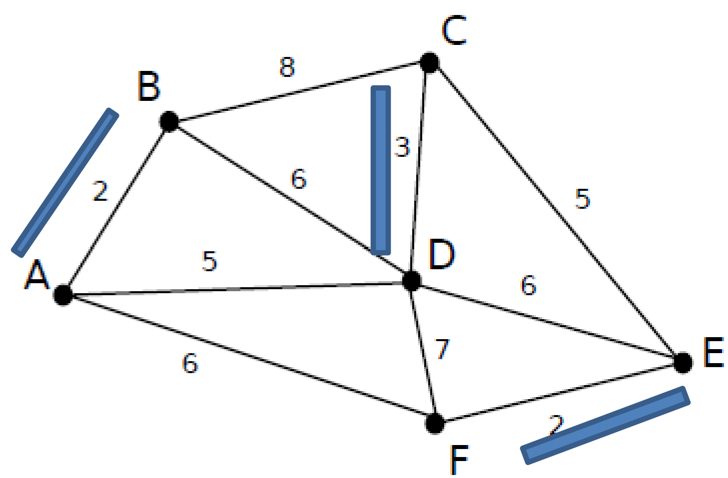
Remark: its complexity is $O(E \log V)$

Minimum Spanning Tree (MST)









Minimum Spanning Tree (MST)