Graphs Theory
Graph representation (2)

University of M’sila
Faculty of Mathematics and informatics
Mathematics department

Speciality:

Applied Mathematics

L3

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ADJACENCY MATRIX

- Let G=(V, E) be an directed graph, the **adjacency matrix** of G $(m_{ij})_{i,j\in[1,n]^2}$ $n=|V|$ is the square matrix

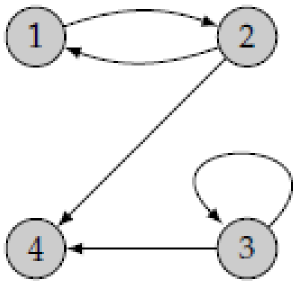
$$(m_{ij})=\begin{cases}1 \text{ if } \exists \text{ edge from } i \text{ to } j\\ 0\text{ , else}\end{cases}$$

- If the graph is undirected the matrix is symmetric.
-

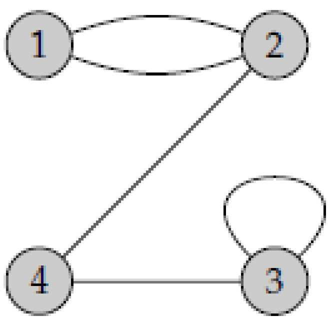
$$(m_{ij})=\begin{cases}k \text{ if } \exists \text{ k edges from } i \text{ to } j\\ 0\text{ , else}\end{cases}$$

ADJACENCY MATRIX

- **Example:**



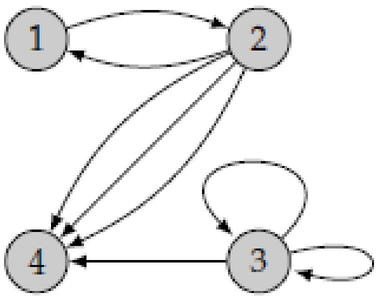
$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



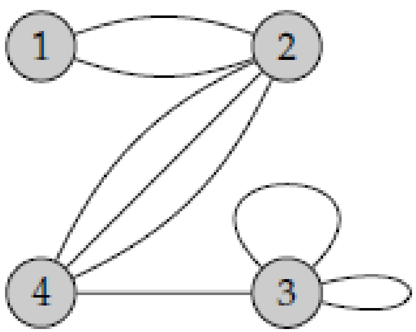
$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

ADJACENCY MATRIX

- **Example:**



$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



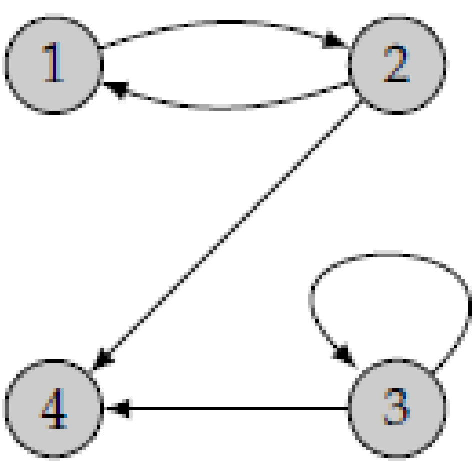
$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 3 & 1 & 0 \end{pmatrix}$$

Operations on ADJACENCY MATRIX

- ❑ **Complexity :** $O(n^2)$
- ❑ **Neighbors (Adjacent vertices)**
- ❑ **Example:**

Predecessor vertices

Successors vertices



$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

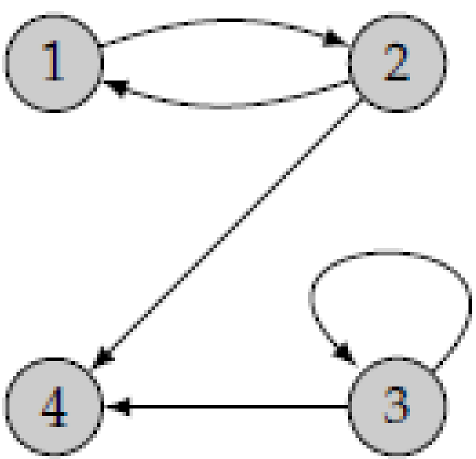
Operations on ADJACENCY MATRIX

- **Degree**

Predecessor vertices

Successors vertices

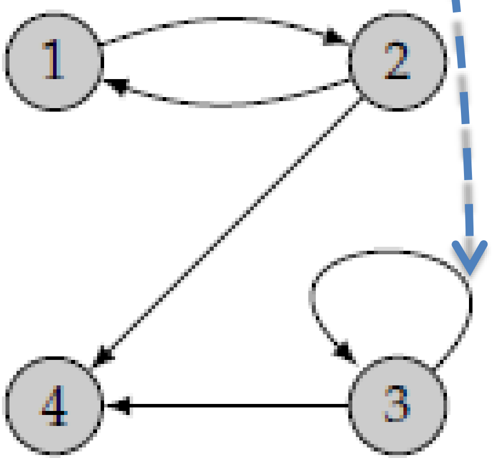
$$\sum$$



$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Operations on ADJACENCY MATRIX

- Loops
- ….

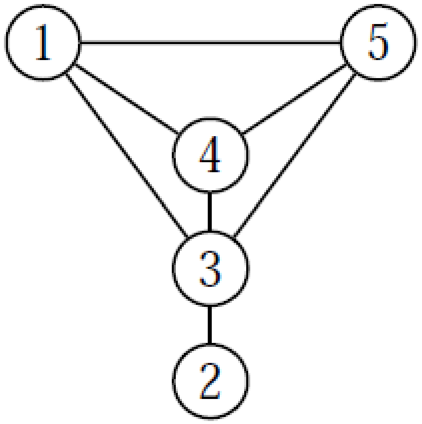


$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Operations on ADJACENCY MATRIX

- ❑ **Square matrix** $n \times n$
- ❑ **1 in the diagonal** $\exists\ \mathrm{loop}$
- ❑ **If the graph is undirected graph the matrix is symmetric.**

Operations on ADJACENCY MATRIX



$$M = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Number of chains have the length 3 between vertex (i) and vertex (j)

$$M^2 = \begin{pmatrix} 3 & 1 & 2 & 2 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 0 & 4 & 2 & 2 \\ 2 & 1 & 2 & 3 & 2 \\ 2 & 1 & 2 & 2 & 3 \end{pmatrix}$$

$$M^3 = \begin{pmatrix} 6 & 2 & 8 & 7 & 7 \\ 2 & 0 & 4 & 2 & 2 \\ 8 & 4 & 6 & 8 & 8 \\ 7 & 2 & 8 & 6 & 7 \\ 7 & 2 & 8 & 7 & 6 \end{pmatrix}$$

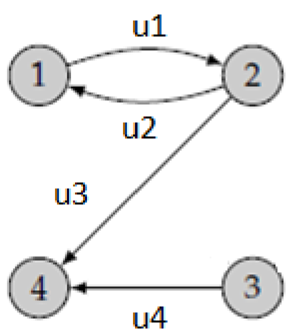
INCIDENCE MATRIX

- Let G=(V, E) be an directed graph, the **incidence matrix** of G $(m_{ij})_{i,j\in[1,n\times m]}$ $n=|V|, \mathrm{m}=|\mathrm{E}|$

$$(m_{ij})=\begin{cases}+1 \text{ if } i \text{ is the start vertex of the edge } j \\ -1 \text{ if } i \text{ is the end vertex of the edge } j \\ 0, \text{ else}\end{cases}$$

INCIDENCE MATRIX

- **Example:**



$$
\begin{array}{ccccc}
 & u1 & u2 & u3 & u4 \\
1 & +1 & -1 & 0 & 0 \\
2 & -1 & +1 & +1 & 0 \\
3 & 0 & 0 & 0 & +1 \\
4 & 0 & 0 & -1 & -1
\end{array}
$$

- **Complexity:**

$$O(n \times m)$$

Adjacency Lists

- Let $G=(V,E)$ be a graph, the representation of G using Adjacency lists consists of an array of n lists each one for one vertex.

$$n=|V|$$

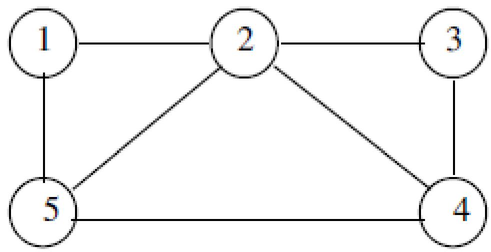
Note: in the case of directed graph this list consists of list of successors/predecessors if the actual vertex.

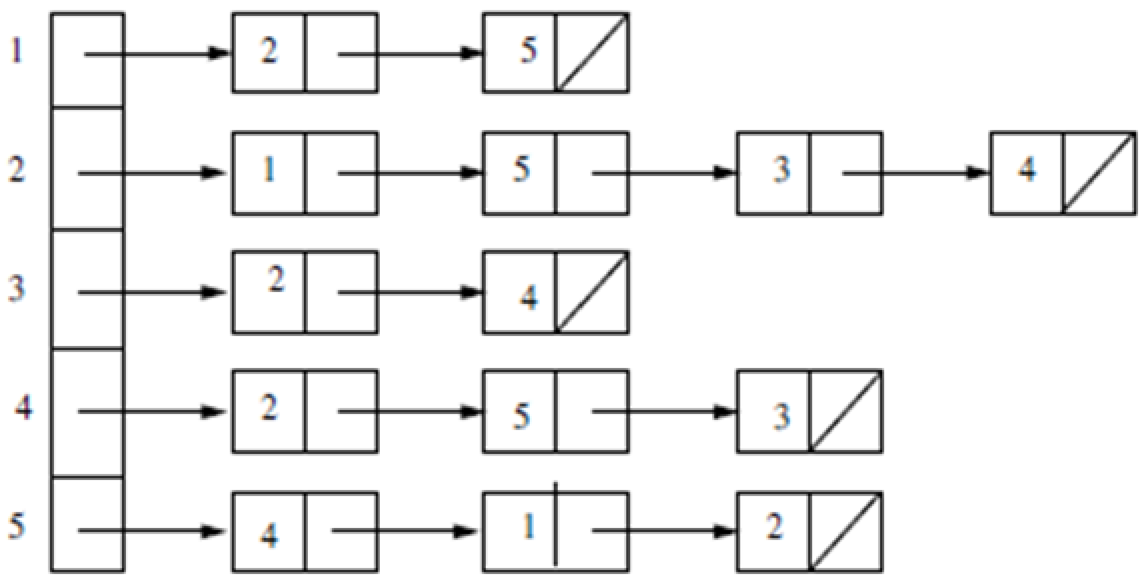
- **Complexity:**

$$O(n+m)$$

Adjacency Lists

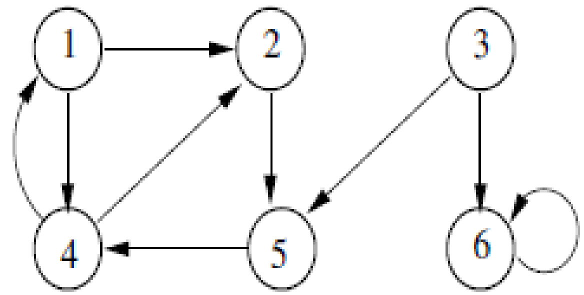
- **Example:**

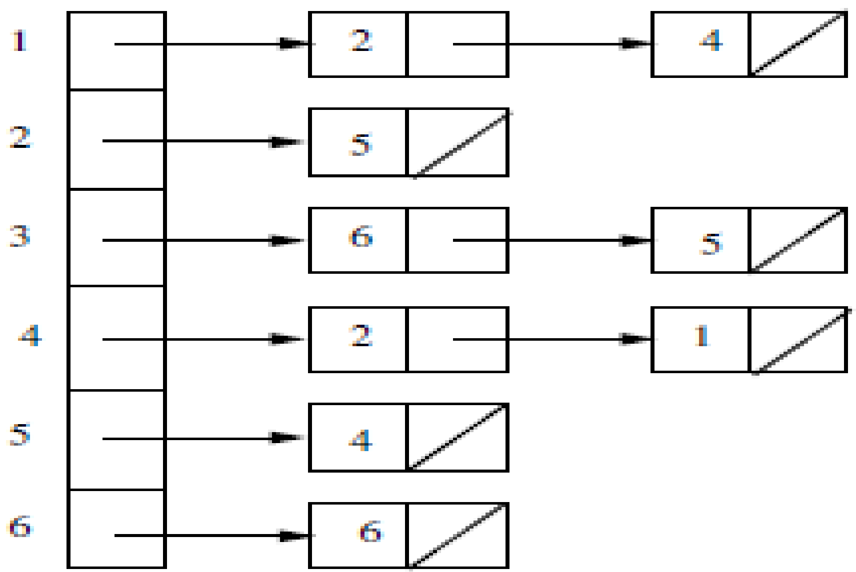




Adjacency Lists

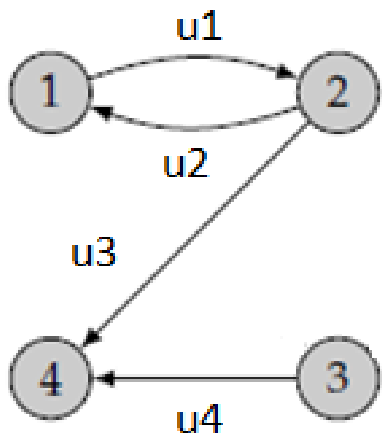
- **Example (List of successors)**





Lists of sorted edges

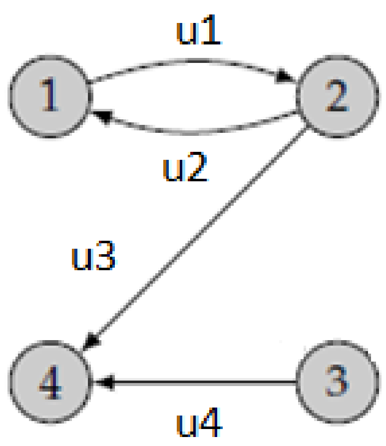
- ❑Arbitrary (no order)



u1	u2	u3	u4
1	2	2	3
2	1	4	4

Lists of sorted edges

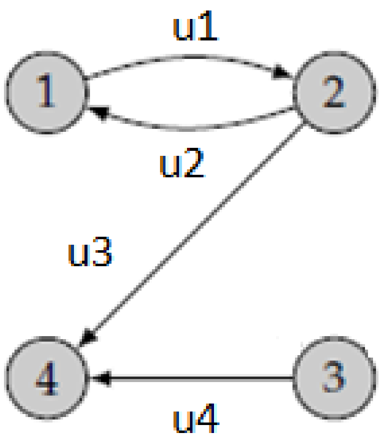
- ❑ **Order according to the start end_point**



u1	u2	u3	u4
1	2	2	3
2	1	4	4

Lists of sorted edges

- Order according to the terminal end_point



u2	u1	u3	u4
2	1	2	3
1	2	4	4