

TD No. 02
Simulation and Software Practice

Exercise 1 :

We consider a congruential generator defined by the relation:

$$\begin{cases} x_n = (9 \cdot x_{n-1} + 1) \mod 32, \\ x_0 = 3. \end{cases}$$

1. Generate the values x_n as well as their normalized versions.
2. Perform the χ^2 test at the 5% significance level to check whether the u_n follow a uniform distribution on $[0, 1[$.

Exercise 2 :

Let R be a random variable with exponential distribution of parameter $\frac{1}{2}$, and Θ a random variable uniformly distributed on $[0, 2\pi]$, independent.

We define:

$$X = \sqrt{R} \cos(\Theta) \quad \text{and} \quad Y = \sqrt{R} \sin(\Theta).$$

1. Prove that X and Y follow independent standard normal distributions with density $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

Exercise 3 :

Use the inversion method to simulate a random variable X following a geometric distribution with parameter $p \in]0, 1[$.

Exercise 4 :

Use the inversion method to simulate a random variable following an exponential distribution with parameter $\lambda > 0$, whose density is given by:

$$e^{-\lambda z} 1_{\{z > 0\}}.$$