Chapter 3

Confidence Interval Estimation

In the previous chapter, we illustrated how a parameter can be estimated from sample data. However, in the study of certain phenomena, point estimates alone often fail to provide information about the precision of the results, since they do not account for errors due to sampling variability. It is therefore essential to understand how accurate a parameter estimate is likely to be.

To assess the degree of confidence that one can have in an estimated value of a parameter, we aim to find an interval that contains the true value of the parameter with a predetermined probability, called the **confidence level**, denoted by $1-\alpha$. Here, α (a positive real number less than 1) represents the **significance level** or **risk level**. This procedure is known as **interval estimation** or, more precisely, **confidence interval estimation**.

A confidence interval should therefore have a high probability of containing the true value of the parameter, while being associated with a small probability of error α.

3.1 Confidence Intervals

Definition 3.1.1. *The **interval estimation** of an unknown parameter θ consists in finding, from a chosen estimator $\hat{\theta}$, an interval*

$$CI = [L, U],$$

such that it is likely that the true value of the parameter lies within this interval. The probability associated with this confidence interval, denoted by $\mathbb{P}(CI)$, is expressed as a percentage $1-\alpha$, where α is the risk level (or significance level).

The probability $1-\alpha$ is called the ***confidence level*** *(or* ***confidence threshold****) :*

$$\begin{aligned}\mathbb{P}(CI) &= \mathbb{P}(L \leq \theta \leq U)\\ &= 1-\alpha,\end{aligned}$$

where $1-\alpha$ represents the probability that the interval contains the true parameter value, and L (respectively U) denote the lower and upper bounds of the confidence interval.

Remark 7. *The confidence level is always associated with the interval itself, not with the unknown parameter θ.*

To determine the confidence interval CI, we usually consider **symmetric risks**, that is :

$$\begin{cases}\mathbb{P}(\theta \leq L) &= \dfrac{\alpha}{2},\\ \mathbb{P}(\theta \geq U) &= \dfrac{\alpha}{2}.\end{cases}$$

3.2 Confidence Interval Estimation of the Mean of a Normal Distribution

The goal is to determine, from the sample mean $\bar{X}$ of a sample drawn from a normal distribution, a confidence interval

$$CI = [L, U]$$

that is likely to contain the true value of the population mean m.

3.2.1 Confidence Interval for the Mean When the Standard Deviation σ Is Known

Consider a random sample $(X_1, X_2, \ldots, X_n)$ from a normal distribution with mean m and variance $Var(X) = \sigma^2$, where σ is known. The sample mean $\bar{X}$ satisfies :

$$\begin{cases}\mathbb{E}(\bar{X}_n) &= m = \hat{m}_n,\\ Var(\bar{X}_n) &= \dfrac{\sigma^2}{n},\end{cases}$$

so that the standard deviation of $\bar{X}_n$ is

$$S_n = \frac{\sigma}{\sqrt{n}}.$$

Hence, $\bar{X} \sim \mathcal{N}\left(m, \frac{\sigma^2}{n}\right)$. We define the standardized variable :

$$Z = \frac{\bar{X} - m}{\frac{\sigma}{\sqrt{n}}} \tag{3.1}$$

which follows the standard normal distribution $Z \sim \mathcal{N}(0, 1)$.

3.2.1.0.1 Problem. We seek bounds L and U such that :

$$\mathbb{P}(L \leq m \leq U) = 1 - \alpha.$$

Assuming symmetric risks, we find from the standard normal distribution table the value $z_{\alpha/2}$ such that :

$$\mathbb{P}(-z_{\alpha/2} \leq Z \leq z_{\alpha/2}) = 1 - \alpha. \tag{3.2}$$

Substituting (3.1) into (3.2), we get :

$$\mathbb{P}\left(-z_{\alpha/2} \leq \frac{\bar{X} - m}{\frac{\sigma}{\sqrt{n}}} \leq z_{\alpha/2}\right) = 1 - \alpha,$$

which can be rewritten as :

$$\mathbb{P}\left(\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq m \leq \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha.$$

Therefore, the $(1 - \alpha)$ confidence interval for m is :

$$CI = \left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right], \tag{3.3}$$

where $1 - \alpha$ is the **confidence level**.

Definition 3.2.1 (Confidence Interval for the Mean with Known Variance)**.** *Let m be the mean of a random sample of size n drawn from a population with known variance σ^2. A $(1 - \alpha)$ confidence interval (CI) for m is given by :*

$$\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq m \leq \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}},$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ quantile of the standard normal distribution, satisfying (3.2).

Example 3.2.2 (Confidence Interval for a Normal Mean with Known Variance)**.** *Consider a sample of $n = 100$ observations from the distribution $\mathcal{N}(m, 1)$. We wish to construct a 95% confidence interval for m, given that $\bar{X} = 2$.*

Since

$$\begin{aligned} \mathbb{P}(Z \le z_{\alpha/2}) &= 1 - \frac{\alpha}{2} \\ &= 1 - \frac{0.05}{2} \\ &= 0.975, \end{aligned}$$

we obtain from standard normal tables that $z_{0.025} = 1.96$.

Thus,

$$\begin{aligned} CI &= \left[2 - 1.96 \cdot \frac{1}{\sqrt{100}},\ 2 + 1.96 \cdot \frac{1}{\sqrt{100}}\right] \\ &= [1.804,\ 2.196]. \end{aligned}$$

It is evident that the higher the confidence level of the interval (i.e., the smaller the value of α), the wider the interval becomes. This is illustrated by the following numerical example.

Example 3.2.3 (Effect of the Confidence Level and Sample Size on the Confidence Interval Width)**.** *Using statistical software (e.g.,* R*), we simulate a sample of size $n = 20$ from a normal distribution $\mathcal{N}(0, 1)$. The sufficient statistic for this model is :*

$$T(\underline{X}) = \sum_{i=1}^{n} X_i,$$

and for this particular sample we obtain $T(\underline{x}) = 8.2314$.

Taking $\alpha = 5\%$, we find (using statistical tables or software) that the $(1-\alpha/2)$-quantile of the standard normal distribution is $z_{\alpha/2} = 1.96$. Thus, the 95% confidence interval $(1 - \alpha = 0.95)$ for the mean μ is :

$$[-0.0267,\ 0.8498].$$

As noted above, increasing the confidence level results in a wider interval. For instance, if we desire a 99% confidence interval (i.e., $\alpha = 1\%$), the $(1 - \alpha/2)$-quantile is $z_{\alpha/2} = 2.5758$ (often rounded to 2.58). Using the same sample values, the 99% confidence interval becomes :

$$[-0.1644,\ 0.9875],$$

which indeed contains the previous 95% interval.

Now suppose we simulate ten additional observations from the same model (so that $n = 30$*), and obtain a new value for the sufficient statistic :* $T(\underline{x}) = 6.2539$*. The 95% confidence interval then becomes :*

$$[-0.1494,\ 0.5663],$$

which clearly illustrates that as the sample size n increases, the confidence interval becomes narrower and more centered around the true parameter value μ *(here,* $\mu = 0$*).*

3.2.2 One-Sided Confidence Limits

The confidence interval given in expression (3.3) simultaneously provides both a lower and an upper confidence limit for the mean m, thus yielding a *two-sided* confidence interval (CI). It is also possible, however, to obtain a *one-sided* confidence limit for m by setting either $L = -\infty$ or $U = +\infty$, and replacing $z_{\alpha/2}$ with z_α.

Definition 3.2.4. *An* ***upper confidence limit*** *for m at the* $(1-\alpha)$ *confidence level is :*

$$m \le \bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}} = U, \tag{3.4}$$

and a ***lower confidence limit*** *for m at the* $(1-\alpha)$ *confidence level is :*

$$m \ge \bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}} = L. \tag{3.5}$$

Exercice 1

The lifetime (in hours) of a 100-watt light bulb is normally distributed with $\sigma = 25$ hours. A random sample of $n = 100$ bulbs has a sample mean lifetime of $\bar{X} = 1140$ hours.

1. Determine a two-sided 95% confidence interval for the mean lifetime of the bulbs.
2. Determine a one-sided lower 95% confidence bound for the mean lifetime of the bulbs.

Solution 1

The random variable T, representing the lifetime of 100 bulbs, follows a normal distribution with unknown mean m and known standard deviation $\sigma = 25$. The confidence level is $1 - \alpha = 0.95$, corresponding to $\alpha = 0.05$.

1. From expression (3.3), the 95% two-sided confidence interval is :
$$\text{CI} = \left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right],$$
where $z_{\alpha/2}$ satisfies (3.2) :
$$\mathbb{P}(Z \leq z_{\alpha/2}) = 1 - \frac{\alpha}{2} = 0.975.$$
From the standard normal distribution, $z_{\alpha/2} = 1.96$. Therefore,
$$\text{CI} = [1140-1.96\times 2.5,\ 1140+1.96\times 2.5] = [1135.1,\ 1144.9]\,\text{hours}.$$

2. From expression (3.5), the 95% one-sided lower confidence bound is :
$$L = \bar{X} - z_{\alpha}\frac{\sigma}{\sqrt{n}},$$
where $\mathbb{P}(Z \leq z_{\alpha}) = 1 - \alpha = 0.95$. From the standard normal table, $z_{\alpha} = 1.645$. Hence,
$$L = 1140 - 1.645 \times 2.5 = 1135.89.$$
Thus, the 95% one-sided lower confidence interval is :
$$[1135.89,\ +\infty).$$

3.2.3 Confidence Interval Estimation of the Mean of a Normal Distribution when the Standard Deviation σ is Unknown

When the value of σ is unknown, it is still possible to determine a confidence interval for the mean m by using a result derived from Fisher's theorem.

We first recall the definitions and properties of several continuous distributions derived from the normal distribution, beginning with the chi-square distribution.

Chi-Square Distribution

Definition 3.2.5. *Let $(X_1, \ldots, X_n)$ be n independent random variables, each following the standard normal distribution $\mathcal{N}(0,1)$. The random variable*

$$X = \sum_{i=1}^{n} X_i^2$$

*is said to follow a **chi-square distribution** with n degrees of freedom. We denote this distribution by $\chi^2(n)$ or χ^2_n. Its probability density function is given by :*

$$f(x;n) = \frac{1}{2^{n/2}\,\Gamma\left(\frac{n}{2}\right)}\, x^{n/2-1}\, e^{-x/2}, \quad x \geq 0,$$

where Γ is the Gamma function defined as

$$\Gamma(t) = \int_0^{+\infty} e^{-y}\, y^{t-1}\, dy.$$

The expected value and variance of a chi-square random variable with n degrees of freedom are given by :

$$\mathbb{E}(X) = n \quad \textit{and} \quad Var(X) = 2n.$$

Normal Approximation to the Chi-Square Distribution

If the sample size $n > 100$, then a random variable X following a chi-square distribution can be approximated by a normal distribution with mean $\mathbb{E}(X) = n$ and variance $Var(X) = 2n$, that is :

$$X \sim \mathcal{N}(n, 2n).$$

Student's *t*-Distribution

Definition 3.2.6. *Let Z and U be two independent random variables such that $Z \sim \mathcal{N}(0,1)$ (standard normal) and $U \sim \chi^2_n$ (chi-square with n degrees of freedom). The random variable*

$$T = \frac{Z}{\sqrt{U/n}}$$

*is said to follow the **Student's t-distribution** with n degrees of freedom, denoted by t_n or T_n. Its probability density function is given by :*

$$f(t;n) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)} \left(1 + \frac{t^2}{n}\right)^{-\frac{n+1}{2}}, \quad t \in \mathbb{R}.$$

Here, Γ denotes Euler's Gamma function, defined as

$$\Gamma(z) = 2\int_0^{+\infty} y^{2z-1}e^{-y^2}\,dy.$$

The expected value and variance of the t-distribution are :

$$\mathbb{E}(T) = \begin{cases} 0, & n > 1, \\ \text{undefined}, & n \leq 1, \end{cases} \qquad Var(T) = \begin{cases} \dfrac{n}{n-2}, & n > 2, \\ \text{undefined}, & n \leq 2. \end{cases}$$

Theorem 4. *Let $X = (X_1, X_2, \ldots, X_n)$ be a sample of independent and identically distributed random variables such that*

$$X_i \sim \mathcal{N}(m, \sigma^2), \quad i = 1, 2, \ldots, n.$$

Define the sample mean and the unbiased sample variance as

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad \tilde{S}_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2.$$

Then :

1. *The statistics $\bar{X}_n$ and $\tilde{S}_n^2$ are independent.*
2. *The statistic $\dfrac{(n-1)\tilde{S}_n^2}{\sigma^2}$ follows a chi-square distribution with $n-1$ degrees of freedom :*
$$\frac{(n-1)\tilde{S}_n^2}{\sigma^2} \sim \chi^2_{n-1}.$$
3. *The statistic $\sqrt{n}\,\dfrac{\bar{X}_n - m}{\tilde{S}_n}$ follows a Student's t-distribution with $n-1$ degrees of freedom :*
$$\sqrt{n}\,\frac{\bar{X}_n - m}{\tilde{S}_n} \sim t_{n-1}.$$

Now, we can determine the confidence interval for the mean m when σ is unknown.

By replacing σ in expression (3.1) with its estimator $\tilde{S}_n$, we obtain

$$\begin{aligned} T &= \frac{\sqrt{n}(\bar{X} - m)}{\tilde{S}_n} \\ &= \sqrt{n-1}\cdot\frac{\bar{X} - m}{S_n}, \end{aligned}$$

where T follows the Student's t-distribution with $(n-1)$ degrees of freedom.

Thus,

$$\begin{aligned}\mathbb{P}\left(|\bar{X}-m| \leq \varepsilon\right) &= \mathbb{P}\left(|T| \leq \sqrt{n}\frac{\varepsilon}{\tilde{S}_n}\right) \\ &= 1-\mathbb{P}\left(|T| > \sqrt{n}\frac{\varepsilon}{\tilde{S}_n}\right),\end{aligned}$$

where $T \sim t_{n-1}$. From the Student's t-distribution table, we find the value $t_{n-1,\alpha/2}$ such that

$$\mathbb{P}(|T| > t_{n-1,\alpha/2}) = \alpha.$$

Consequently,

$$t_{n-1,\alpha/2} = \sqrt{n}\frac{\varepsilon}{\tilde{S}_n},$$

which gives

$$\varepsilon = \frac{\tilde{S}_n}{\sqrt{n}}\, t_{n-1,\alpha/2}.$$

Finally, the confidence interval becomes

$$\mathrm{CI} = \left[\bar{X} - \frac{\tilde{S}_n}{\sqrt{n}}\, t_{n-1,\alpha/2},\ \bar{X} + \frac{\tilde{S}_n}{\sqrt{n}}\, t_{n-1,\alpha/2}\right].$$

Remark 8. *The statistic $\tilde{S}_n$ represents the **unbiased estimator** of the population standard deviation, obtained from the unbiased sample variance*

$$\tilde{S}_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2.$$

It is used here because, unlike the biased estimator $S_n^2 = \frac{1}{n}\sum(X_i-\bar{X})^2$, it ensures correct inference properties and validity of the Student's t-distribution result.

Remark 9. *The quantile $t_{n-1,\alpha/2}$ of order $1-\frac{\alpha}{2}$ is the value found in the column labeled $P=\alpha$ and the row corresponding to $n-1$ degrees of freedom in the Student's t-distribution table.*

Definition 3.2.7. *If $\bar{X}$ and $\tilde{S}_n$ are the sample mean and the unbiased sample standard deviation of a sample from a normal distribution with unknown* *standard deviation σ, then the $(1-\alpha)$ confidence interval for the population mean m is given by :*

$$\text{CI} = \left[\bar{X} - \frac{\tilde{S}_n}{\sqrt{n}}\, t_{n-1,\alpha/2},\ \bar{X} + \frac{\tilde{S}_n}{\sqrt{n}}\, t_{n-1,\alpha/2} \right]. \tag{3.6}$$

Example 3.2.8. *Let $n = 20$, $\bar{X} = 64.2$, $\tilde{S}_n = 5.15$, and $\alpha = 5\%$. From the Student's t-distribution table, we find $t_{19,\,0.025} = 2.093$, so*

$$\text{CI} = [\,64.2 - \tfrac{5.15}{\sqrt{20}} \times 2.093,\ 64.2 + \tfrac{5.15}{\sqrt{20}} \times 2.093\,] = [61.8,\ 66.7].$$

Remark 10. *A confidence interval that is too wide is of little practical interest, since the true parameter value cannot be determined precisely. Therefore, it is sometimes necessary to accept a relatively higher risk level α in order to obtain a more usable (narrower) confidence interval.*

Definition 3.2.9. *Let $\bar{X}$ and $\tilde{S}_n$ be the sample mean and unbiased sample standard deviation of a sample from a normal distribution with unknown population standard deviation σ.*

An ***upper confidence limit*** *at level $(1-\alpha)$ for the population mean m is :*

$$m \leq \bar{X} + \frac{\tilde{S}_n}{\sqrt{n}}\, t_{n-1,\alpha} = UCL,$$

and a ***lower confidence limit*** *at level $(1-\alpha)$ for the population mean m is :*

$$m \geq \bar{X} - \frac{\tilde{S}_n}{\sqrt{n}}\, t_{n-1,\alpha} = LCL.$$

Remark 11. *The quantile $t_{n-1,\alpha}$ of order $1-\alpha$ is the value found in the column labeled $P = \alpha$ and the row corresponding to $n-1$ degrees of freedom in the Student's t-distribution table.*

3.3 Confidence Interval for the Variance

We use Fisher's Theorem 4 to derive a function of the observations $X_1, X_2, \ldots, X_n$ and of σ^2 whose probability distribution does not depend on either m or σ^2.

Since

$$\frac{n\,S_n^2}{\sigma^2} \sim \chi^2_{n-1},$$

we can write, for $0 < a < b$,

$$\mathbb{P}\left(a < \frac{n\,S_n^2}{\sigma^2} < b\right) = \mathbb{P}\left(\frac{n\,S_n^2}{b} < \sigma^2 < \frac{n\,S_n^2}{a}\right) = 1 - \alpha.$$

To balance the tail risks, we choose a and b such that

$$F_{\chi^2_{n-1}}(b) = 1 - \frac{\alpha}{2} \quad \text{and} \quad F_{\chi^2_{n-1}}(a) = \frac{\alpha}{2}.$$

The quantile $z_{n-1,\alpha}$ of the chi-square distribution satisfies

$$\mathbb{P}(Z > z_{n-1,\alpha}) = \alpha.$$

Hence, for $b = z_{n-1,\alpha/2}$ and $a = z_{n-1,1-\alpha/2}$, we obtain

$$\mathbb{P}\left(\frac{n\,S_n^2}{b} < \sigma^2 < \frac{n\,S_n^2}{a}\right) = 1 - \alpha.$$

Finally, the $(1-\alpha)$ confidence interval for σ^2 is :

$$\begin{aligned} IC &= \left[\frac{n\,S_n^2}{z_{n-1,\alpha/2}},\ \frac{n\,S_n^2}{z_{n-1,1-\alpha/2}}\right], \\ &= \left[\frac{(n-1)\,\tilde{S}_n^2}{z_{n-1,\alpha/2}},\ \frac{(n-1)\,\tilde{S}_n^2}{z_{n-1,1-\alpha/2}}\right]. \end{aligned}$$

Example 3.3.1. *Let $n = 20$, $\tilde{S}_n^2 = 26.5$, and $\alpha = 0.05$. From the chi-square distribution table, we find :*

$$z_{19,0.025} = 32.85, \quad z_{19,0.975} = 8.91.$$

Hence,

$$IC = [15.3,\ 56.6].$$

This interval is quite wide compared to that for the mean m, reflecting that variance estimation is inherently less precise.

Theorem 5 (Central Limit Theorem)**.** *Let X be a random variable with mean m and standard deviation σ. When the sample size n is large, the sampling distribution of the sample mean $\bar{X}$ can be approximated by a normal distribution with mean m and standard deviation $\frac{\sigma}{\sqrt{n}}$. Consequently, the standardized variable*

$$Z = \frac{\bar{X} - m}{\sigma/\sqrt{n}}$$

is approximately distributed as $\mathcal{N}(0,1)$.

3.4 Confidence Interval for a Population Proportion

In many applications, it is necessary to construct confidence intervals for a population proportion. Suppose a random sample of size n is drawn from a very large (possibly infinite) population. If $X \le n$ observations in this sample belong to a class of interest, then

$$\hat{p} = \frac{X}{n}$$

is a point estimator of the population proportion p belonging to that class. Here, n and p are the parameters of a binomial distribution.

When $np \ge 5$ and $n(1-p) \ge 5$, the binomial distribution can be approximated by a normal distribution with mean np and variance $np(1-p)$. Therefore, the standardized variable

$$\begin{aligned} Z &= \frac{X - np}{\sqrt{np(1-p)}} \\ &= \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \end{aligned}$$

is approximately distributed as $\mathcal{N}(0,1)$.

Definition 3.4.1. *The* $(1-\alpha)$ *confidence interval for the population proportion p is given by :*

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \tag{3.7}$$

and satisfies

$$\mathbb{P}\left(\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 1-\alpha,$$

where $1-\alpha$ *is the confidence level and* $z_{\alpha/2}$ *is the upper* $\alpha/2$ *quantile of the standard normal distribution, i.e.*

$$\mathbb{P}(Z \le z_{\alpha/2}) = 1 - \frac{\alpha}{2}.$$

Example 3.4.2. *In a random sample of* 100 *tires,* 10 *exhibit a surface finish rougher than permitted by specifications. Consequently, the point estimate of the proportion of defective tires is :*

$$\hat{p} = \frac{10}{100} = 0.1.$$

A two-sided 95% *confidence interval for* p *is obtained from expression* (3.7) *:*

$$\hat{p} - z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Substituting numerical values gives

$$0.1 - 1.96\sqrt{\frac{0.1 \times 0.9}{100}} \leq p \leq 0.1 + 1.96\sqrt{\frac{0.1 \times 0.9}{100}},$$

so that

$$IC = [0.0412,\ 0.1588].$$

Definition 3.4.3. *An upper confidence limit at level* $(1-\alpha)$ *for the proportion* p *is :*

$$p \leq \hat{p} + z_{\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = LS,$$

and a lower confidence limit at the same level is :

$$p \geq \hat{p} - z_{\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = LI.$$

3.5 Examples

Example 3.5.1. *A sample* $n = 100$ *from* $\mathcal{N}(m, 1)$ *gives* $\overline{X} = 2$*. For* $\alpha = 0.05$, $z_{0.025} = 1.96$.

$$CI = [1.804,\ 2.196].$$

Example 3.5.2. *The lifetime of light bulbs* $\sim \mathcal{N}(m, \sigma^2)$ *with* $\sigma = 25$, $n = 100$, $\overline{X} = 1140$.

$$CI = [1135.1,\ 1144.9].$$