

Faculty of Mathématiques and Informatics	Date : 11.01.2026	Class : Master II, ISSI
Department of Computer Science	Duration: 02 h	Course: Introduction to AI

END OF SEMESTER EXAM

EXERCISE 01	Course Questions	15 Minutes	3.0 points
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1. Give the dual equivalent of each of the following expressions: *(01 pt)*

$$\neg \exists x \varphi \quad \neg \forall x \varphi \quad \neg (\varphi \vee \psi) \quad \neg (\varphi \wedge \psi)$$

2. What are applications of AI in the following domains : NLP, Handwriting Recognition *(02 pts)*

EXERCISE 02	First-Order Predicate Logic	35 Minutes	4.0 points
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1. Conversion into First-Order Predicate Logic of the Following Natural Language Sentences:
(0.5 point/ assertion) :

1. There is a module that no student understands.
2. Although nobody makes any noise, Omar cannot concentrate.
3. All students except Nassim are present.
4. Some students master mathematics and some others do not master it.
5. All students belonging to a specialty have a fundamental teaching unit that they must acquire.
6. The rule of the game is that a team can neither win nor lose against itself.

2. Transformation of the Two Assertions Expressed in First-Order Predicate Logic Using the Duality Principle of Quantifiers: *(01 pt)*

- a. $\exists x \text{Follow}(x, \text{News})$
- b. $\forall x \text{Master}(x, \text{Language}) \Leftrightarrow$

EXERCISE 03	Machine Learning	40 Min	8.5 points
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Given a Dataset of 2300 Biscuits. We have four types of biscuits: Petit Four, Cookies, Maxon, and Kool. For each biscuit, we have the following characteristics:

- The biscuit is small or large
- It is sweet or less sweet
- It contains chocolate or vanilla

Our dataset is presented as follows:

Type	Small	Large	Sweet	Not sweet	chocolate	Vanilla	Total
Petit four	350	250	400	200	400	200	600
Cookies	150	350	200	300	100	400	500
Maxon	300	100	250	150	200	200	400
Kool	500	300	350	450	500	300	800
Total	1300	1000	1200	1100	1200	1100	2300

Let the biscuit be characterized by : $B = \langle \text{Large}, \text{Sweet}, \text{Vanilla} \rangle$

1. Apply the Naive Bayes Algorithm to Determine the Type of This Biscuit

Convolutional Neural Networks (CNNs) have been applied in the field of image processing and computer vision CV for long time.

1. Which of the following is important application in image processing? **(0.75 pts)**
 - A. Feature extraction and image classification
 - B. 3D image reconstruction
 - C. Augmented reality gaming
 - D. Image style transfer
2. Explain briefly explain the principle of convolution operation is and its the role in CNNs. **(0.75 pts)**
3. Draw (or cite) a simple CNN architecture and explain its main components. **(01 pt)**
4. Let be an input image size of 200x200 pixels, using a filter size of 5x5, a stride of 1, and no padding. Calculate the size of the output Image. **(0.5 pt)**
5. What challenges might you encounter during the training process of deep CNNs? **(01 pt)**
 - A. Vanishing or exploding gradients
 - B. Overfitting
 - C. High computational resource requirements
 - D. Class imbalance
6. Give some well-known CNNs architectures **(0.5 pts)**

The figure bellow represents an image IM1 of size 4x4 and a kernel (a filter) K1 of size 3x3

35	30	90	50
45	0	2	5
0	30	25	0
255	100	65	0

Une Image 4X4

1	1	2
0	0	0
-1	-1	-2

Un filtre 3X3

7. Do the convolution of the image matrix IM1 and the kernel K1 and give the obtained image IM2. **(01 pt)**

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END OF SEMESTER EXAM: A STANDARD CORRECTION

EXERCISE 01	Course Questions	15 Minutes	3.0 points
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1. Give the equivalent expression of each of the following expressions: (01 pt)

- $\neg \exists x \varphi$ is equivalent to $\forall x \neg \varphi$
- $\neg \forall x \varphi$ is equivalent to $\exists x \neg \varphi$
- $\neg (\varphi \vee \psi)$ is equivalent to $\neg \varphi \wedge \neg \psi$
- $\neg (\varphi \wedge \psi)$ is equivalent to $\neg \varphi \vee \neg \psi$

2. What are applications of AI in the following domains : NLP, Handwriting Recognition (02 pts)

In NLP :

- Understanding of natural language.
- QA systems
- Automatic Translation.
- Text Summarizing.
- Smart Chatbots
- Human-machine interaction (replace high-level languages).

In Handwriting Recognition

- Archeology
- Credit Fraud detection for checks.
- Police investigations.
- Correction of exam papers, ...
- Educational Systems

EXERCISE 02	First-Order Predicate Logic	35 Minutes	4.0 points
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1. Conversion into First-Order Predicate Logic of the Following Natural Language Sentences: (0.5 point/ assertion) :

1. There is a module that no student understands.

$$\exists x, \forall y (\text{Module}(x) \wedge \text{Student}(y) \wedge \neg \text{Understand}(y, x))$$

2. Although nobody makes any noise, Omar cannot concentrate.

$$\forall x (\text{Person}(x) \wedge \neg \text{Make}(x, \text{Noise}) \Rightarrow \neg \text{Concentrate}(\text{Omar}))$$

3. All students except Nassim are present.

$$\forall x (\text{Student}(x) \wedge (x \neq \text{Nassim}) \wedge \text{Present}(x))$$

4. Some students master mathematics and some others do not master it.

$$\exists x (\text{Student}(x) \wedge \text{Master}(x, \text{Mathematics}) \wedge (\exists y, \text{Student}(y) \wedge \neg \text{Master}(y, \text{Mathematics})))$$

$$\exists x, \exists y (\text{Student}(x) \wedge \text{Student}(y) \wedge \text{Master}(x, \text{Mathematics}) \wedge \neg \text{Master}(y, \text{Mathematics}))$$

5. All students belonging to a specialty have a fundamental teaching unit that they must acquire.

$$\forall x, \exists y, \exists z (\text{Student}(x) \wedge \text{Specialty}(y) \wedge \text{FundamentalUnit}(z) \wedge \text{Belong}(x, y) \wedge \text{Acquire}(x, z))$$

6. The rule of the game is that a team can neither win nor lose against itself.

$$\forall x (\text{Team}(x) \wedge \neg \text{Win}(x, x) \wedge \neg \text{Lose}(x, x))$$

2. Transformation of the Two Assertions Expressed in First-Order Predicate Logic Using the Duality Principle of Quantifiers: (01 pt)

c. $\exists x \text{Follow}(x, \text{News}) \Leftrightarrow \neg \forall x \neg \text{Follow}(x, \text{News})$

d. $\forall x \text{Master}(x, \text{Language}) \Leftrightarrow \neg \exists x \neg \text{Master}(x, \text{Language})$

Given a Dataset of 2300 Biscuits. We have four types of biscuits: Petit Four, Cookies, Maxon, and Kool. For each biscuit, we have the following characteristics:

- The biscuit is small or large
- It is sweet or less sweet
- It contains chocolate or vanilla

Our dataset is presented as follows:

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Kool	500	300	350	450	500	300	800
Total	1300	1000	1200	1100	1200	1100	2300

Let the biscuit be characterized by : $B = \langle \text{Large, Sweet, Vanilla} \rangle$

1. Apply the Naive Bayes Algorithm to Determine the Type of This Biscuit

To classify this biscuit into one of the types (**Petit Four, Cookies, Maxon, Kool**), we need to compute the following four probabilities: **(01 pt)**

- $P(\text{Petit Four}|\text{large, sweet, vanilla})$: The probability that the biscuit is a Petit Four given that it is $\langle \text{large, sweet, vanilla} \rangle$.
- $P(\text{Cookies}|\text{large, sweet, vanilla})$: The probability that the biscuit is a Cookies given that it is $\langle \text{large, sweet, vanilla} \rangle$
- $P(\text{Maxon}|\text{large, sweet, vanilla})$: The probability that the biscuit is a Maxon given that it is $\langle \text{large, sweet, vanilla} \rangle$
- $P(\text{Kool}|\text{large, sweet, vanilla})$: The probability that the biscuit is a Kool given that it is $\langle \text{large, sweet, vanilla} \rangle$

The type of the “**unknown**” biscuit to be classified is the one with the **highest probability**. Using the following Bayes formula:

$$P(X|C_i) = \frac{P(C_i|X_j) \cdot P(C_i)}{P(X)}$$

We must calculate: **(01 pt)**

$$P(\text{petit four} | \text{Large, Sweet, Vanilla}) = \frac{P(\text{Large}|\text{petit four}) \cdot P(\text{sweet}|\text{petit four}) \cdot P(\text{vanilla}|\text{petit four})}{P(\text{Large}) \cdot P(\text{Sweet}) \cdot P(\text{Vanilla})} \cdot P(\text{petit four}) \quad (1)$$

$$P(\text{Cookies} | \text{Large, Sweet, Vanilla}) = \frac{P(\text{Large}|\text{Cookies}) \cdot P(\text{Sweet}|\text{Cookies}) \cdot P(\text{Vanilla}|\text{Cookies})}{P(\text{Large}) \cdot P(\text{Sweet}) \cdot P(\text{Vanilla})} \cdot P(\text{Cookies}) \quad (2)$$

$$P(\text{Maxon} | \text{Large, Sweet, Vanilla}) = \frac{P(\text{Large}|\text{Maxon}) \cdot P(\text{Sweet}|\text{Maxon}) \cdot P(\text{Vanilla}|\text{Maxon})}{P(\text{Large}) \cdot P(\text{Sweet}) \cdot P(\text{Vanilla})} \cdot P(\text{Maxon}) \quad (3)$$

$$P(\text{Kool} | \text{Large, Sweet, Vanilla}) = \frac{P(\text{Large}|\text{Kool}) \cdot P(\text{Sweet}|\text{Kool}) \cdot P(\text{Vanilla}|\text{Kool})}{P(\text{Large}) \cdot P(\text{Sweet}) \cdot P(\text{Vanilla})} \cdot P(\text{Kool}) \quad (4)$$

Then we calculate : **(01 pt)**

$$P(\text{petit four}) = \frac{\text{Cardinal}(\text{petit four})}{\text{Total biscuit}} = \frac{600}{2300} = \frac{6}{23} = 0.261$$

$$P(\text{Cookies}) = \frac{\text{Cardinal}(\text{Cookies})}{\text{Total biscuit}} = \frac{500}{2300} = \frac{5}{23} = 0.217$$

$$P(\text{Maxon}) = \frac{\text{Cardinal}(\text{Maxon})}{\text{Total biscuit}} = \frac{400}{2300} = \frac{4}{23} = 0.174$$

$$P(\text{Kool}) = \frac{\text{Cardinal}(\text{Kool})}{\text{Total biscuit}} = \frac{800}{2300} = \frac{8}{23} = 0.3478$$

We can calculate or neglect the following probabilities :

$$P(\text{Large}) = \frac{\text{Cardinal}(\text{Large})}{\text{Total biscuit}} = \frac{1000}{2300} = \frac{10}{23} = 0.435$$

$$P(\text{Sweet}) = \frac{\text{Cardinal}(\text{Sweet})}{\text{Total biscuit}} = \frac{1200}{2300} = \frac{12}{23} = 0.522$$

$$P(\text{Vanilla}) = \frac{\text{Cardinal}(\text{Vanilla})}{\text{Total biscuit}} = \frac{1100}{2300} = \frac{11}{23} = 0.478$$

For (1), we calculate : **(0.75 Pt)**

$$P(\text{Large}|\text{petit four}) = \frac{\text{Cardinal}(\text{Petit four AND Large})}{\text{Cardinal}(\text{Petit four})} = \frac{250}{600} = \frac{5}{12} = 0.417$$

$$P(\text{Sweet}|\text{Petit four}) = \frac{\text{Cardinal}(\text{Petit four AND Sweet})}{\text{Cardinal}(\text{Petit four})} = \frac{400}{600} = \frac{2}{3} = 0.667$$

$$P(\text{Vanilla}|\text{Petit four}) = \frac{\text{Cardinal}(\text{petit four AND vanilla})}{\text{Cardinal}(\text{petit four})} = \frac{200}{600} = \frac{1}{3} = 0.333$$

Thus, the final result of (1) is : **(0.5 pt)**

$$P(\text{petit four} | \text{Large, Sweet, Vanilla}) = \frac{0.417 \cdot 0.667 \cdot 0.333}{0.435 \cdot 0.522 \cdot 0.478} \cdot 0.261 = \frac{0.0242}{0.1085} = \mathbf{0.223}$$

For (2), we calculate : **(0.75 Pt)**

$$P(\text{Large}|\text{Cookies}) = \frac{\text{Cardinal}(\text{Cookies AND Large})}{\text{Cardinal}(\text{Cookies})} = \frac{350}{500} = \frac{35}{50} = 0.7$$

$$P(\text{Sweet}|\text{Cookies}) = \frac{\text{Cardinal}(\text{Cookies AND Sweet})}{\text{Cardinal}(\text{Cookies})} = \frac{200}{500} = \frac{2}{5} = 0.4$$

$$P(\text{Vanilla}|\text{Cookies}) = \frac{\text{Cardinal}(\text{Cookies AND Vanilla})}{\text{Cardinal}(\text{Cookies})} = \frac{400}{500} = \frac{4}{5} = 0.8$$

Thus, the final result of (2) is : **(0.5 pt)**

$$P(\text{Cookies}|\text{Large, Sweet, Vanilla}) = \frac{0.7 \cdot 0.4 \cdot 0.8}{0.435 \cdot 0.522 \cdot 0.478} \cdot 0.217 = \frac{0.048608}{0.1085} = \mathbf{0.448}$$

For (3), we calculate: **(0.75 Pt)**

$$P(\text{Large}|\text{Maxon}) = \frac{\text{Cardinal}(\text{Maxon AND Large})}{\text{Cardinal}(\text{Maxon})} = \frac{100}{400} = \frac{1}{4} = 0.25$$

$$P(\text{Sweet}|\text{Maxon}) = \frac{\text{Cardinal}(\text{Maxon AND Sweet})}{\text{Cardinal}(\text{Maxon})} = \frac{250}{400} = \frac{5}{8} = 0.625$$

$$P(\text{Vanilla}|\text{Maxon}) = \frac{\text{Cardinal}(\text{Maxon AND Vanilla})}{\text{Cardinal}(\text{Maxon})} = \frac{200}{400} = \frac{2}{4} = 0.5$$

Thus, the final result of (3) is : **(0.5 pt)**

$$P(\text{Maxon} | \text{Large, Sweet, Vanilla}) = \frac{0.25 \cdot 0.625 \cdot 0.5}{0.435 \cdot 0.522 \cdot 0.478} \cdot 0.174 = \frac{0.013594}{0.1085} = \mathbf{0.125}$$

For (4), we calculate : **(0.75 Pt)**

$$P(\text{Large}|\text{Kool}) = \frac{\text{Cardinal}(\text{Kool AND Large})}{\text{Cardinal}(\text{Kool})} = \frac{300}{800} = \frac{3}{8} = 0.375$$

$$P(\text{Sweet}|\text{Kool}) = \frac{\text{Cardinal}(\text{Kool AND Sweet})}{\text{Cardinal}(\text{Kool})} = \frac{350}{800} = \frac{35}{80} = 0.4375$$

$$P(\text{Vanilla}|\text{Kool}) = \frac{\text{Cardinal}(\text{Kool AND Vanilla})}{\text{Cardinal}(\text{Kool})} = \frac{300}{800} = \frac{3}{8} = 0.375$$

Thus, the final result of (4) is : **(0.5 pt)**

$$P(\text{Kool} | \text{Large, Sweet, Vanilla}) = \frac{0.375 \cdot 0.4375 \cdot 0.375}{0.435 \cdot 0.522 \cdot 0.478} \cdot 0,3478 = \frac{0,0213978}{0.1085} = \mathbf{0.197}$$

Finally, we will have :

$$P(\text{petit four} | \text{Large, Sweet, Vanilla}) = \mathbf{0.223}$$

$$P(\text{Cookies} | \text{Large, Sweet, Vanilla}) = \mathbf{0.448}$$

$$P(\text{Maxon} | \text{Large, Sweet, Vanilla}) = \mathbf{0.125}$$

$$P(\text{Kool} | \text{Large, Sweet, Vanilla}) = \mathbf{0.197}$$

We conclude that the probability that the biscuit will be Cookies $P(\text{Cookies} | \text{Large, Sweet, Vanilla})$ is greater than the other probabilities $\Rightarrow$ The given biscuit is classified in the class Cookies **(0.5 pt)**

EXERCISE 04

Deep Learning

30 Min

5.5 points

Convolutional Neural Networks (CNNs) have been applied in the field of image processing and computer vision CV for decades.

1. Which of the following is important application in image processing? (0.75 pts)

- A. Feature extraction and image classification
- B. 3D image reconstruction
- C. Augmented reality gaming
- D. Image style transfer

A, B, D are important applications in CNNs, but C is not

2. Explain briefly explain the principle of convolution operation is and its the role in CNNs. (0.75 pts)

The Convolution is a mathematical operation that consists to multiply successively a matrice by a set of filters where each filter (also called kernel) is a two-dimensional 2D matrix containing integer values.

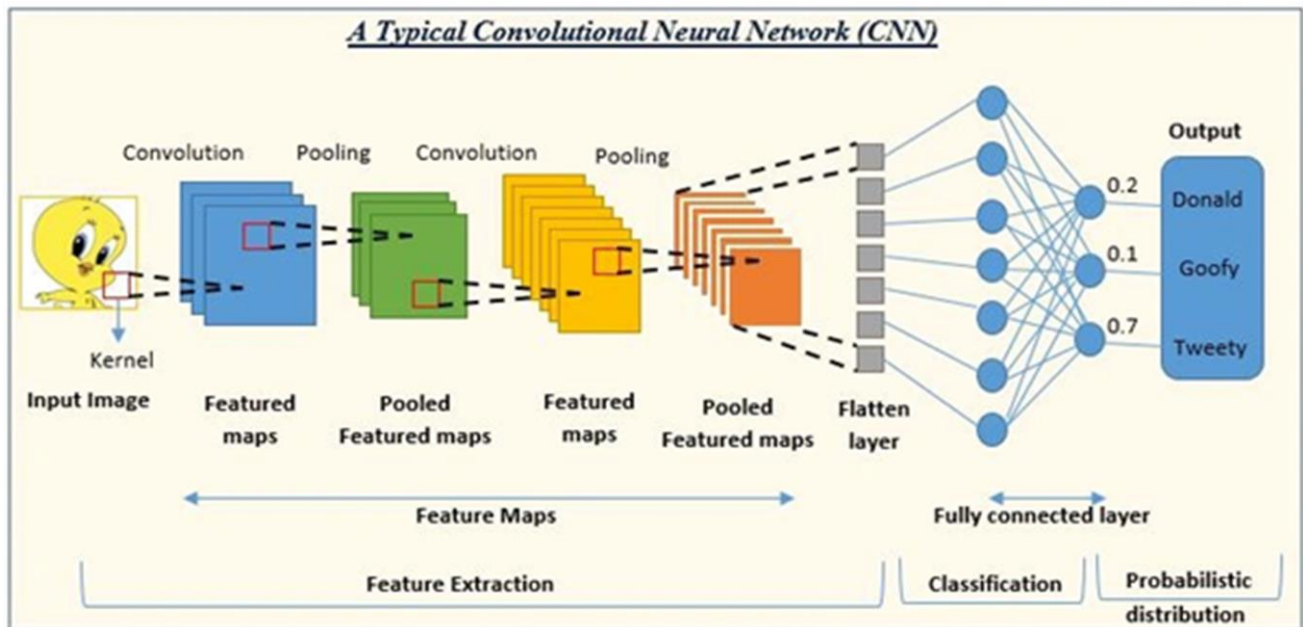
Main Objectives :

- Reduce the size of the input image $\Rightarrow$ Reduce the memory space and processing time.
- Extract the most significant (relevant) feature (pixel) in the image without losing information.

3. Draw (or cite) a simple CNN architecture and explain its main components. (01 pt)

This CNN consists of :

1. Convolutional layers
2. Followed by an activation function layer (e.g., ReLU)
3. Pooling layers.
4. A Fully Connected Layer



4. Let be an input image size of 200x200 pixels, using a filter size of 5x5, a stride of 1, and no padding. Calculate the size of the output Image. (0.5 pt)

The size of the output image can be calculated using the formula:

$$\text{Output size} = [(\text{Input size} - \text{Filter size} + 2 * \text{Pad}) / \text{Stride}] + 1.$$

Input Image Size = 200 , Filter Size = 5 ; Pad=0 ; Stride = 1

Thus, for a 200x200 input image, a 5x5 filter, Padding = 0, and a stride of 1, the output image size is

$$[(200 - 5 + 2 * 0) / 1] + 1 = 196 \times 196 \text{ pixels}.$$

5. What challenges might you encounter during the training process of deep CNNs? (01 pt)

- A. Vanishing or exploding gradients
- B. Overfitting
- C. High computational resource requirements
- D. Class imbalance

Encountered challenges during the training process of deep CNNs are : A, B, C ; D

6. Give some well-known CNNs architectures (0.5 pts)

LeNet, AlexNet, VGG, ResNet, etc

The figure bellow represents an image IM1 of size 4x4 and a kernel (a filter) K1 of size 3x3

35	30	90	50
45	0	2	5
0	30	25	0
255	100	65	0

Une Image 4X4

1	1	2
0	0	0
-1	-1	-2

Un filtre 3X3

7. Do the convolution of the image matrix IM1 and the kernel K1 and give the obtained image IM2. (01 pt)

